Topic: Differentiation 1 & 2 in Book 1 (Topics 50 to 59)

Q1. Differentiate $f(x) = 2x^2 - 3x + 1$ from first principles. **Q2.** Differentiate $\sqrt{4-3x}$ with respect to x. **Ans:** $\frac{-3}{2\sqrt{4-3x}}$ **Q3.** Given that $y = \ln(\frac{3+x}{\sqrt{9-x^2}})$ for $x > 3$, find $\frac{dy}{dx}$ and express your answer in the form $\frac{a}{b-x^c}$ where $a, b, c \in N$. **Ans:** $\frac{3}{9-x^2}$	**Q4.** Differentiate the following (i) $\tan^{-1} 5x$ at x = $\frac{1}{10}$ (ii) $\sqrt{\frac{1+3x}{x-1}}$ at x = 5 (ii) $\sqrt{1+3x}$ (iii) $\frac{e^{3x}}{x}$ (iv) $\log_e(x^3+1)^3$ (v) $x^3.e^x$ (vi) $e^{\cos x}$ at x = 0 (vii) $\cos^3(5x^2)$ **Ans:** (i) 4 (ii) $-\frac{1}{16}$ (iii) $\frac{e^{3x}(3x-1)}{x^2}$ (iv) $\frac{9x^2}{1+x^3}$ (v) $e^x(x^3+3x^2)$ (vi) 0 (vii) $-30x\cos^2(5x^2).\sin(5x^2)$
Q5. Let f(x) = $x^3 + kx^2 - 4$, where $k > 0$. Show that the coordinates of the local minimum and local maximum of f(x) are (0, -4) and $(-\frac{2k}{3}, \frac{4k^3-108}{27})$.	**Q6.** Find the coordinates of the local maximum and local minimum points of the curve $y = (x^2-2)e^{-2x}$. **Ans:** Min: $(-1, -e^2)$ Max: $(2, \frac{2}{e^4})$
Q7. Let $f(x) = \frac{3}{x-2}, x \neq 2$. (i) Show that the graph of f(x) is always decreasing. (ii) Given that 3x + 4y + k = 0 is a tangent to the graph of f(x), where $k \in Z$, find the two possible values of k. (ii) **Ans:** k = 6, -18	**Q8.** Oil is dripping onto a surface at the rate of $\frac{\pi}{10}$ cm^3/s and forms a circular film which may be considered to have a uniform depth of 0.1cm. Find the rate at which the radius of the circular film is increasing when the radius is 5cm. **Ans:** 0.1 cm/s
Q9. An object's distance from a fixed point is s = $12 + 24t - 3t^2$. Find the speed of the object when t = 3secs. **Ans:** 6 m/s	**Q10.** Find the equation of the tangent to the curve $5x^2 + 5y^2 = 13$ at the point (3, 1). **Ans:** 3x + y - 10 = 0
Q11. The radius of the base of a right circular cylinder is r cm and its height is 2r cm. (i) Find the rate at which the volume is increasing, when the radius is 2cm and in increasing at a rate of 0.25 cm/s. (ii) Find the rate at which the total surface area is increasing when the radius is 5cm and the volume is increasing at a rate of 5π cm^3/s. **Ans:** (i) 6π cm^3/s (ii) 2π cm^2/s	**Q12.** A square of side length x cm is cut from each of the corners of a rectangular piece of cardboard with dimensions 15cm by 24cm. The cardboard is then folded to form an open box of depth x cm. (i) Show that the volume of the box is $4x^3 - 78x^2 + 360x$ cm^3. (ii) Find the value of x for which the volume of the box is a maximum, and the max volume. **Ans:** (ii) x = 3cm, Vmax = 486 cm^3
Q13. Let $y = \frac{x}{x-3}$. Prove that the graph has no turning points and no points of inflection.	**Q14.** The radius of a sphere is increasing at 2cm/s. Find the rate at which its volume is increasing when $r = \frac{1}{2} cm$. **Ans:** 2π cm^3/s
Q15. Find the equation of the tangent to the curve $f(x) = \sqrt{x^2-3}$ at the point (2, 1). **Ans:** 2x - y - 3 = 0	**Q16.** A lump of modelling clay of volume 72 cm^3 is moulded into the shape of a rectangular box with sides of length x cm, 2x cm and y cm. Find the minimum surface area of this rectangular box. **Ans:** 108 cm^2
Q17. Find the slope of the curve $x^2 + y^2 - 6x + 3y + 8 = 0$ at the point (-1, 2). Ans: $\frac{8}{7}$	**Q18.** If y = $4x^3 - 6x^2$ show that $x^2.\frac{d^2y}{dx^2} - 2x.\frac{dy}{dx} - 12x^2 = 0$.