Question 6 **(30 marks)**

(a) $h(x) = x^2 + bx - 12$, where $x \in \mathbb{R}$ and b is a constant.

Find the value of b for which $x - 4$ is a factor of $h(x)$.

$$h(4) = 0$$
$$(4)^2 + b(4) - 12 = 0$$
$$4b = -4$$
$$b = -1$$

(b) Write the following expression as a single fraction in terms of t:

$$\frac{4}{2t+1} - \frac{7}{12t}$$

$$\frac{4}{2t+1} - \frac{7}{12t} = \frac{4(12t) - 7(2t+1)}{(2t+1)(12t)}$$
$$= \frac{48t - 14t - 7}{(2t+1)(12t)}$$
$$= \frac{34t - 7}{(2t+1)(12t)}$$

Question 1 **(30 marks)**

(a) Solve the following equation for $n \in \mathbb{N}$:

$$n - 3 = \sqrt{3n+1}$$

$$(n-3)^2 = \left(\sqrt{3n+1}\right)^2$$
$$n^2 - 6n + 9 = 3n + 1$$
$$n^2 - 9n + 8 = 0$$
$$(n-8)(n-1) = 0$$
$$n = 8, n = 1$$

Answer: $n = 8$ [as $n = 1$ gives $-2 = \sqrt{4}$]

(c) Solve the following simultaneous equations for $x, y, w \in \mathbb{Z}$:

$$x + 2y = 143$$
$$y + 3w = -74$$
$$4x + 5w = 4$$

1:	$x + 2y = 143$
2x(-2):	$-2y - 6w = 148$
4:	$x - 6w = 291$
3:	$4x + 5w = 4$
4x(4):	$4x - 24w = 1164$
5:	$29w = -1160$
So	$w = -40$
4:	$x - 6(-40) = 291$
So	$x = 51$
1:	$51 + 2y = 143$
So	$y = 46$

Question 1 **(30 marks)**

(a) Find the two values of $m \in \mathbb{R}$ for which $|5 + 3m| = 11$.

Method 1:

$5 + 3m = 11$	$5 + 3m = -11$
$3m = 6$	$3m = -16$
$m = 2$	$m = -\frac{16}{3}$

Method 2:

$$(5 + 3m)^2 = 11^2$$
$$25 + 30m + 9m^2 = 121$$
$$9m^2 + 30m - 96 = 0$$
$$3m^2 + 10m - 32 = 0$$
$$(3m + 16)(m - 2) = 0$$
$$m = -\frac{16}{3}, \quad m = 2$$

(c) $x^2 - px + 1$ is a factor of $x^3 - 2x - 3r$, where $p, r \in \mathbb{R}$ and $p < 0$.

Find the value of p and the value of r.

Method 2

$$(x^2 - px + 1)(x + k) = x^3 - 2x - 3r$$
$$x^3 - px^2 + x + kx^2 - pkx + k = x^3 - 2x - 3r$$
$$x^3 + (k - p)x^2 + (1 - pk)x + k = x^3 - 2x - 3r$$

$k - p = 0$	$1 - pk = -2$	$k = -3r$
$k = p$	$1 - p^2 = -2$	$r = -\frac{k}{3}$
	$p^2 = 3$	$= -\frac{-\sqrt{3}}{3}$
	$p = -\sqrt{3}$ $[p < 0]$	$= \frac{\sqrt{3}}{3}$

(b) For the real numbers h, j, and k:

$$\frac{1}{h} = \frac{k}{j + k}$$

Express k in terms of h and j.

$$j + k = hk$$
$$k - hk = -j$$
$$k(1 - h) = -j$$
$$k = -\frac{j}{1-h} \quad \text{or} \quad k = \frac{j}{h-1}$$

(b) $f(x) = 2x^3 - 21x^2 + 40x + 63$, where $x \in \mathbb{R}$.

(i) $x + 1$ is a factor of $f(x)$. Find the three values of x for which $f(x) = 0$.

$$\begin{array}{r l}
 & 2x^2 - 23x + 63 \\
x + 1 & \overline{)\,2x^3 - 21x^2 + 40x + 63} \\
 & \underline{2x^3 + 2x^2} \\
 & \quad -23x^2 + 40x + 63 \\
 & \quad \underline{-23x^2 - 23x} \\
 & \qquad 63x + 63 \\
 & \qquad \underline{63x + 63} \\
 & \qquad\quad 0
\end{array}$$

$$(x + 1)(2x^2 - 23x + 63) = 0$$
$$(x + 1)(2x - 9)(x - 7) = 0$$
$$x = -1, 4{\cdot}5, \text{ or } 7$$

(ii) The areas of the three regions **K**, **L**, and **N** give the following three equations (including the equation from **part (b)(i)**):

$$\begin{aligned} 4a + 3b + 3c &= 807 \\ 28a + 9b + 3c &= 879 \\ 76a + 15b + 3c &= 663 \end{aligned}$$

Solve these equations to find the values of a, b, and c.

(2) − (1): $24a + 6b = 72$ **...(4)**

(3) − (2): $48a + 6b = -216$ **...(5)**

(5) − (4): $24a = -288$

So $a = -12$

(4): $24(-12) + 6b = 72$

$6b = 360$ so $b = 60$

(1): $4(-12) + 3(60) + 3c = 807$

$3c = 675$ so $c = 225$

Question 1 **(30 marks)**

(a) Find the two values of $m \in \mathbb{Z}$ for which the following equation in x has exactly **one** solution:

$$3x^2 - mx + 3 = 0$$

$b^2 - 4ac = 0$

$m^2 - 4(3)(3) = 0$

$m^2 = 36$

$m = \pm 6$

(b) Explain why the following equation in x has **no** real solutions:

$$(2x + 3)^2 + 7 = 0$$

$(2x+3)^2 \geq 0$ for all $x \in \mathbb{R}$

So $(2x+3)^2 + 7 \geq 7 > 0$

OR

$(2x+3)^2 = -7$

$2x + 3 = \pm\sqrt{-7}$, which is not real

OR

$4x^2 + 12x + 9 + 7 = 0$

$4x^2 + 12x + 16 = 0$

$b^2 - 4ac = 12^2 - 4(4)(16) < 0$,

so no real roots

(c) (i) Show that $x = -1$ is **not** a solution of $3x^2 + 2x + 5 = 0$.

(i) $3(-1)^2 + 2(-1) + 5$

$= 3 - 2 + 5 = 6 \neq 0$

(ii) Find the **remainder** when $3x^2 + 2x + 5$ is divided by $x + 1$.

That is, find the value of c when $3x^2 + 2x + 5$ is written in the form

$$3x^2 + 2x + 5 = (x+1)(ax+b) + c$$

where $a, b, c \in \mathbb{Z}$.

(ii) $(x+1)(ax+b) + c$

$= ax^2 + (a+b)x + b + c = 3x^2 + 2x + 5$

So $a = 3$,

$3 + b = 2$, so $b = -1$

$-1 + c = 5$, so $c = 6$.

Remainder, $c =$____________

Question 2 **(30 marks)**

(a) Given that $x = -3$ is a solution to $|x + p| = 5$, find the two values of p, where $p \in \mathbb{Z}$.

$|-3 + p| = 5$ *[LPC]*

$9 - 6p + p^2 = 25$ *[MPC]*

$p^2 - 6p - 16 = 0$

$(p+2)(p-8) = 0$ *[HPC]*

$p = -2$ or $p = 8$

OR

$|-3 + p| = 5$

$-3 + p = 5$ **or** $-3 + p = -5$

$p = 8$ **or** $p = -2$

(b) $(x + 4)$ is a factor of $f(x) = x^3 + qx^2 - 22x + 56$, where $x \in \mathbb{R}$ and $q \in \mathbb{Z}$.

Show that $q = -5$, **and** find the three roots of $f(x)$.

Show

$$\begin{array}{r|l} & x^2 - 9x + 14 \\ x+4 & \overline{x^3 - 5x^2 - 22x + 56} \\ & \underline{x^3 + 4x^2} \\ & \quad -9x^2 - 22x + 56 \\ & \quad \underline{-9x^2 - 36x} \\ & \qquad 14x + 56 \\ & \qquad \underline{14x + 56} \\ & \qquad\quad 0 \end{array}$$

Remainder $= 0$, $\therefore q = -5$

$x^3 - 5x^2 - 22x + 56 = 0$

$x^2(x+4) - 9x(x+4) + 14(x+4) = 0$

$(x^2 - 9x + 14)(x+4) = 0$

$(x-2)(x-7)(x+4) = 0$

Roots $= (-4,\ 2,\ 7)$

Question 1 **(25 marks)**

(a) $f(x) = x^2 + 5x + p$ where $x \in \mathbb{R}$, $-3 \leq p \leq 8$, and $p \in \mathbb{Z}$.

(i) Find the value of p for which $x + 3$ is a factor of $f(x)$.

$f(-3) = 0$

$f(-3) = -3^2 + 5(-3) + p = 0$

$9 - 15 + p = 0$

$p = 6$

(ii) Find the value of p for which $f(x)$ has roots which differ by 3.

$\alpha, \alpha + 3 =$ roots

$\alpha + \alpha + 3 = -5$

$2\alpha = -8$

$\alpha = -4$

and $\alpha + 3 = -1$

$p = (-1)(-4) = 4$

(b) Find the range of values of x for which $|2x + 5| - 1 \leq 0$, where $x \in \mathbb{R}$.

$(2x + 5)^2 \leq 1$

$4x^2 + 20x + 25 \leq 1$

$4x^2 + 20x + 24 \leq 0$

$x^2 + 5x + 6 \leq 0$

$(x + 2)(x + 3) \leq 0$

$x = -2, x = -3$

$-3 \leq x \leq -2$

Question 3 **(25 marks)**

(a) Factorise fully: $3xy - 9x + 4y - 12$.

$(3x + 4)(y - 3)$

(b) Solve the equation $\frac{3}{2x+1} + \frac{2}{5} = \frac{2}{3x-1}$ where $x \neq -\frac{1}{2}, \frac{1}{3}$, and $x \in \mathbb{R}$.

$$\frac{3}{2x+1} + \frac{2}{5} = \frac{2}{3x-1}$$

$$\frac{15 + 2(2x+1)}{5(2x+1)} = \frac{2}{3x-1}$$

$$\frac{4x+17}{10x+5} = \frac{2}{3x-1}$$

$$(4x+17)(3x-1) = 2(10x+5)$$

$$12x^2 + 47x - 17 = 20x + 10$$

$$12x^2 + 27x - 27 = 0$$

$$4x^2 + 9x - 9 = 0$$

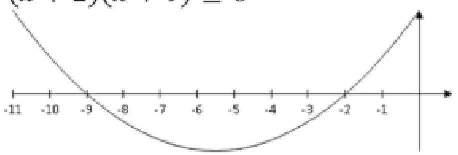
$$(x+3)(4x-3) = 0$$

$$x = -3 \text{ or } x = \frac{3}{4}$$

(b) Solve the inequality $\frac{2x-3}{x+2} \geq 3$, where $x \in \mathbb{R}$ and $x \neq -2$.

$$\frac{2x-3}{x+2} \geq 3 \qquad \times\ (x+2)^2$$

$$(2x-3)(x+2) \geq 3(x+2)^2$$
$$2x^2 + x - 6 \geq 3x^2 + 12x + 12$$
$$x^2 + 11x + 18 \leq 0$$
$$(x+2)(x+9) \leq 0$$



$$-9 \leq x < -2$$

Question 1 **(25 marks)**

(a) Solve the simultaneous equations.

$$\begin{aligned} 2x + 3y - \ \ z &= -4 \\ 3x + 2y + 2z &= 14 \\ x \qquad - 3z &= -13 \end{aligned}$$

$$\begin{aligned} (i)\quad 2x + 3y - z &= -4 \qquad \times (2) \\ (ii)\quad 3x + 2y + 2z &= 14 \qquad \times (-3) \end{aligned}$$

$$\begin{aligned} 4x + 6y - 2z &= -8 \\ -9x - 6y - 6z &= -42 \end{aligned}$$

$$\begin{aligned} -5x - 8z &= -50 \\ (iii)\quad x - 3z &= -13 \qquad \times (5) \\ -5x - 8z &= -50 \\ 5x - 15z &= -65 \\ -23z &= -115 \\ z &= 5 \\ \Rightarrow x &= 2 \\ \Rightarrow y &= -1 \qquad \{2, -1, 5\} \end{aligned}$$

Question 2 **(25 marks)**

(a) Find the range of values of x for which $|x-4| \geq 2$, where $x \in \mathbb{R}$.

$$x^2 - 8x + 16 \geq 4$$

$$x^2 - 8x + 12 \geq 0$$

$$(x-2)(x-6) \geq 0$$

$$x = 2 \qquad x = 6$$

$$\{x | x \leq 2\} \cup \{x | x \geq 6\}$$

(b) Solve the simultaneous equations:

$$x^2 + xy + 2y^2 = 4$$
$$2x + 3y = -1.$$

$$x = \frac{-3y-1}{2}$$

$$\left(\frac{-3y-1}{2}\right)^2 + \left(\frac{-3y-1}{2}\right)(y) + 2y^2 = 4$$

$$11y^2 + 4y - 15 = 0$$

$$(11y+15)(y-1) = 0$$

$$y = \frac{-15}{11} \quad or \quad y = 1$$

$$x = \frac{-3\left(\frac{-15}{11}\right)-1}{2} \quad or \; x = \frac{-3(1)-1}{2}$$

$$x = \frac{17}{11} \quad or \; x = -2$$

Question 2 **(25 marks)**

(a) Find the set of all real values of x for which $2x^2 + x - 15 \geq 0$.

$$f(x) = 2x^2 + x - 15 = (2x-5)(x+3)$$

$$(2x-5)(x+3) = 0$$

$$\Rightarrow x = \frac{5}{2} \text{ or } x = -3$$

(i): $x \geq -3$ and $x \geq \frac{5}{2} \Rightarrow x \geq \frac{5}{2}$

(ii): $x \leq -3$ and $x \leq \frac{5}{2} \Rightarrow x \leq -3$

Solution Set: $\{x \mid x \leq -3\} \cup \{x \mid x \geq \frac{5}{2}\}$

Question 5 **(25 marks)**

(a) Solve the equation $x = \sqrt{x+6}$, $x \in \mathbb{R}$.

$$x = \sqrt{x+6}$$
$$\Rightarrow x^2 = x + 6$$
$$\Rightarrow x^2 - x - 6 = 0$$
$$\Rightarrow (x+2)(x-3) = 0$$
$$\Rightarrow x = -2, \quad x = 3$$

$x = -2$: $\quad -2 \neq \sqrt{-2+6} = \sqrt{4} = 2$ ✗

$x = 3$: $\quad 3 = \sqrt{3+6} = \sqrt{9} = 3$ ✓

Question 2 **(25 marks)**

Solve the equation $x^3 - 3x^2 - 9x + 11 = 0$.

Write any irrational solution in the form $a + b\sqrt{c}$, where $a, b, c \in \mathbb{Z}$.

$f(x) = x^3 - 3x^2 - 9x + 11$

$f(1) = 1^3 - 3(1)^2 - 9 + 11 = 0$

$\Rightarrow x = 1$ is a solution.

$(x-1)$ is a factor

$$\begin{array}{r|l} & x^2 - 2x \quad -11 \\ \hline x-1 & x^3 - 3x^2 - 9x + 11 \\ & x^3 - x^2 \\ \hline & -2x^2 - 9x + 11 \\ & -2x^2 + 2x \\ \hline & -11x + 11 \\ & -11x + 11 \\ \hline \end{array}$$

or

$$(x-1)(x^2 + Ax - 11) = x^3 - 3x^2 - 9x + 11$$
$$\Rightarrow x^3 + Ax^2 - x - x^2 - Ax + 1 = x^3 - 3x^2 - 9x + 11$$
$$\Rightarrow A - 1 = -3$$
$$\Rightarrow A = -2$$

Hence, other factor is $x^2 - 2x - 11$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)} = \frac{2 \pm \sqrt{48}}{2} = \frac{2 \pm 4\sqrt{3}}{2} = 1 \pm 2\sqrt{3}$$

Solutions: $\{1,\ 1 + 2\sqrt{3},\ 1 - 2\sqrt{3}\}$

(b) Solve the simultaneous equations;

$$x + y + z = 16$$
$$\frac{5}{2}x + y + 10z = 40$$
$$2x + \frac{1}{2}y + 4z = 21.$$

$$\begin{array}{l} x+y+z=16 \\ \frac{5}{2}x+y+10z=40 \end{array} \Rightarrow \begin{array}{l} 2x+2y+2z=32 \\ 5x+2y+20z=80 \\ \hline 3x \quad +18z=48 \end{array}$$

$$\begin{array}{l} x+y+z=16 \\ 4x+y+8z=42 \\ \hline 3x \quad +7z=26 \end{array}$$

$$\begin{array}{l} 3x+18z=48 \\ 3x\ +7z=26 \\ \hline 11z=22 \Rightarrow z=2 \end{array}$$

$$3x\ +7z=26 \Rightarrow 3x+7(2)=26 \Rightarrow 3x=12 \Rightarrow x=4$$

$$x+y+z=16 \Rightarrow 4+y+2=16 \Rightarrow y=10$$

(b) Find the set of all real values of x for which $\frac{2x-5}{x-3} \leq \frac{5}{2}$.

Multiply across by $2(x-3)^2$, which is non-negative:

$$2(x-3)(2x-5) \leq 5(x-3)^2$$
$$4x^2 - 22x + 30 \leq 5x^2 - 30x + 45$$
$$0 \leq x^2 - 8x + 15$$
$$0 \leq (x-5)(x-3)$$
$$x \geq 5 \text{ or } x < 3.$$

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Question 1 **(25 marks)**

(a) Solve the simultaneous equations:

$$a^2 - ab + b^2 = 3$$
$$a + 2b + 1 = 0$$

$$a = -2b - 1$$

$$(-2b-1)^2 + (2b+1)b + b^2 = 3$$
$$7b^2 + 5b - 2 = 0$$
$$(7b-2)(b+1) = 0$$
$$b = \tfrac{2}{7} \quad \text{or } b = -1$$
$$a = \tfrac{-11}{7} \quad \text{or } a = 1$$

Solution: $\{b = \frac{2}{7} \text{ and } a = \frac{-11}{7}\}$ or $\{b = -1 \text{ and } a = 1\}$.

(c) Solve the equation $x^2 - 2\sqrt{3}x - 9 = 0$, giving your answers in the form $a\sqrt{3}$, where $a \in \mathbb{Q}$.

$$x = \frac{2\sqrt{3} \pm \sqrt{\left(-2\sqrt{3}\right)^2 - 4(1)(-9)}}{2(1)} = \frac{2\sqrt{3} \pm \sqrt{48}}{2}$$
$$= \frac{2\sqrt{3} \pm 4\sqrt{3}}{2}$$
$$= \sqrt{3} \pm 2\sqrt{3}$$

$$x = -\sqrt{3} \text{ or } x = 3\sqrt{3}$$

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