# Equations of Motion, Velocity/Time Graph Problems:

- $\underline{\text{Q1.}}$  A lorry starts from rest with uniform acceleration. In the eighth second of its motion, it travels 5.625~m.
  - (i) Find the acceleration of the lorry.
  - (ii) Find the distance travelled by the lorry in 20 s from rest.
- Q2. A tram starts from rest at one station and comes to a stop  $1 \, min$  later at the next station. It accelerates uniformly at  $2 \, m/s^2$  until it reaches a speed of  $20 \, m/s$ . It travels at this constant speed for a certain time until it decelerates back to rest at  $4 \, m/s^2$ . Find the average velocity of the tram over the whole journey.
- Q3. A lorry accelerates uniformly along a straight level road from rest at  $0.5 \, m/s^2$ , until it has travelled  $256 \, m$ . It then travels at the constant speed reached for  $800 \, m$ , before braking and decelerating uniformly to rest at  $2 \, m/s^2$ .
  - (i) Sketch a velocity/time graph of the motion of the lorry.
  - (ii) Find the maximum speed reached by the lorry.
  - (iii) Find the total time for the journey.
  - (iv) Find the total distance travelled in kilometres.
- Q4. Two points P and Q lie on a straight level road. A car passes point P with a constant speed of  $20 \, m/s$  and continues at this speed for 12 seconds. The car then accelerates uniformly for 5 seconds to a speed of  $30 \, m/s$ . Finally, the car decelerates uniformly from  $30 \, m/s$  to rest at point Q. The car travels  $75 \, m$  while decelerating. Sketch a velocity/time graph for the motion, and hence find:
  - (i) The acceleration
  - (ii) The deceleration
  - (iii) |PQ|, the distance from P to Q
  - (iv) The average speed of the car as it travels from P to Q.
- Q5. A racing car can accelerate at  $6 m/s^2$  and decelerate at  $15 m/s^2$ . Find the shortest time in which it can travel 1 km from rest to rest.
- Q6. A car starts from rest and accelerates with constant acceleration a. It then travels with constant speed v for time  $t_2$ , and then comes to rest again with constant deceleration a. The total distance travelled is s and the total time taken is t.
  - (i) Show that  $v = \frac{a}{2}(t t_2)$ .
  - (ii) Show that the time for which it travelled at constant speed is  $\sqrt{\frac{at^2-4s}{a}}$ .
- Q7. A lift descends in a lift shaft, first with uniform acceleration 2f followed by uniform deceleration 3f. It starts and finishes at rest. If the height of the lift shaft is h and the time taken is t, show that  $h = \frac{3}{5}ft^2$ .
- Q8. A train accelerates uniformly from rest to a speed of v m/s with uniform acceleration a  $m/s^2$ . It then decelerates uniformly to rest with uniform retardation 3a  $m/s^2$ . The total distance travelled is s metres. If the average speed for the whole journey is  $\sqrt{\frac{s}{2}}$ , find the value of a.
- Q9. The maximum acceleration of a car is  $2 m/s^2$ , and its maximum retardation is  $4 m/s^2$ . Its maximum speed is 40 m/s. Thinking about the shape of graph required in each case find:
  - (i) The minimum time for the car to travel 1 km from rest to rest
  - (ii) The minimum time for the car to travel 384 m from rest to rest.

#### Successive Posts Problems:

- Q10. A particle moving in a straight line with constant acceleration takes 3 s and 5 s to cover two successive distances of 1 m. Find the acceleration and the initial velocity.
- Q11. A stone is dropped from the top of a tower. In the last second of its motion, it falls through a distance which is  $\frac{1}{4}$  of the height of the tower. Find the height of the tower to the nearest metre.

## Overtaking/Colliding Problems:

- Q12. A motorbike passes point A at a constant speed of  $18 \, m/s$ . A second motorbike passes point B at the same instant at a speed of  $10 \, m/s$ , and accelerating at  $2 \, m/s^2$ .
  - (i) How long does it take the second bike to overtake the first bike, and at what distance from point A does this take place?
  - (ii) What is the furthest distance the first bike gets ahead of the second bike?
- Q13. At a certain instant in a cycle race, Brian is 80 m behind Alan. Alan is cycling at a constant speed of 15 m/s. Brian is cycling at 10 m/s at that instant and is accelerating at  $1 m/s^2$ .
  - (i) If Alan is only ??? m from the finish line, show that Brian does not pass him before the finish.
  - (ii) How long after Alan finishes does Brian finish?

## Gravity Problems:

- Q14. A stone is thrown up from the top of a cliff with a velocity of 20 m/s. It lands in the water at the bottom of the cliff 5.5 s later. Find the height of the cliff.
- Q15. A balloon rises from the ground with an acceleration of  $0.25 \, m/s^2$ .
  - (i) How far has it risen, and what is its velocity, 20 s later?
  - (ii) If at this instant a stone is dropped from the balloon, how long will it take to reach the ground?
- Q16. A stone is dropped from the top of a building while at the same time a second stone is thrown up from the base of the building at  $30 \, m/s$ . The two stones pass each other after 4 seconds. Find the height of the building.
- Q17. A stone is thrown up with a velocity of 25 m/s. Three seconds later a second stone is thrown up from the same point with a velocity of 20 m/s. At what height do the stones collide?
- Q18. A particle falls from rest at a point A under gravity. After it has fallen a distance a, another particle is given a downward speed  $\sqrt{8ga}$  from the same starting point A. Show that the two particles collide. Find the distance from A at which they collide.

#### Answers:

Q1. (i) $0.75  m/s^2$ (ii) $150  m$	Q2. 17.5 m/s		Q3. (ii) 16 m/s (iii) 90 s (iv) 1.12 km	
Q4. (i) $2 m/s^2$ (ii) $6 m/s^2$	<u>Q5.</u> 21.6 <i>s</i>		$Q8.\frac{4}{3}$	Q9. (i) 40 s (ii) 24 s
(iii) 440 m (iv) 20 m/s			3	
$Q10\frac{1}{30} m/s^2, \frac{23}{60} m/s$	Q11. 273 m	Q12. (i) 8 s,	144 m (ii) 16 m	Q13. (ii) 0.17 s
Q14. 38.225 m	Q15. (i) 50 m, 5 m/s (ii) 2.72 s		<b>Q16</b> . 120 m	<b>Q17</b> . 17.5 m
<u>Q18.</u> $\frac{9a}{4}$				

## Past Exam Questions:

### 2010 Q1

A car is travelling at a uniform speed of 14 ms<sup>-1</sup> when the driver notices a 1. (a) traffic light turning red 98 m ahead.

> Find the minimum constant deceleration required to stop the car at the traffic light,

- (i) if the driver immediately applies the brake
- if the driver hesitates for 1 second before applying the brake. (ii)
- A particle passes P with speed 20 ms<sup>-1</sup> and moves in a straight line to Q with (b) uniform acceleration.

In the first second of its motion after passing P it travels 25 m.

In the last 3 seconds of its motion before reaching Q it travels  $\frac{13}{20}$  of |PQ|.

Find the distance from P to Q.

### 2011 Q1

A particle is released from rest at A and falls vertically 1. (a) passing two points B and C.

> It reaches B after t seconds and takes  $\frac{2t}{7}$  seconds to fall from B to C, a distance of 2.45 m.

Find the value of t.

**(b)** A car accelerates uniformly from rest to a speed v in  $t_1$  seconds. It continues at this constant speed for t seconds and then decelerates uniformly to rest in  $t_2$  seconds.

The average speed for the journey is  $\frac{3v}{4}$ .

- Draw a speed-time graph for the motion of the car.
- Find  $t_1 + t_2$  in terms of t. (ii)
- part (ii).

# 2015 Q1

A particle starts from rest and moves with constant acceleration. 1.

> If the particle travels 39 m in the seventh second, find the distance travelled in the tenth second.

A train of length 66.5 m is travelling with uniform acceleration  $\frac{4}{7}$  m s<sup>-2</sup>.

It meets another train of length 91 m travelling on a parallel track in the opposite direction with uniform acceleration  $\frac{8}{7}$  m s<sup>-2</sup>.

Their speeds at this moment are 18 m s<sup>-1</sup> and 24 m s<sup>-1</sup> respectively.

- Find the time taken for the trains to pass each other.
- Find the distance between the trains 1 second later. (ii)

#### 2016 Q1

- 1. (a) A car has an initial speed of u m s<sup>-1</sup>. It moves in a straight line with constant acceleration f for 4 seconds. It travels 40 m while accelerating. The car then moves with uniform speed and travels 45 m in 3 seconds. It is then brought to rest by a constant retardation 2f.
  - (i) Draw a speed-time graph for the motion.
  - (ii) Find the value of u.
  - (iii) Find the total distance travelled.
  - (b) A particle is projected vertically upwards with a velocity of u m s<sup>-1</sup>. After an interval of 2t seconds a second particle is projected vertically upwards from the same point and with the same initial velocity.

They meet at a height of h m.

Show that 
$$h = \frac{u^2 - g^2 t^2}{2g}$$
.

#### 2018 Q1(b)

(b) A car C moves with uniform acceleration a from rest to a maximum speed u. It then travels at uniform speed u.

Just as car C starts, it is overtaken by a car D moving in the same direction with constant speed  $\frac{3u}{4}$ .

Car C catches up with car D when car C has travelled a distance d.

- (i) Show that, at the instant car C catches up with car D, car C has been travelling with speed u for a time  $\frac{4d}{3u} \frac{u}{a}$ .
- (ii) Find d in terms of u and a.

#### 2024 Q1(c)

John is cycling on a straight horizontal road at a constant velocity of 10 m s<sup>-1</sup> and is 21 m behind another cyclist, Kevin, who is cycling at a constant velocity of 4 m s<sup>-1</sup> in the same direction. John begins to accelerate at 2 m s<sup>-2</sup>. One second later, Kevin begins to accelerate at  $4 \text{ m s}^{-2}$ .

Calculate the times when John and Kevin overtake each other.

## 2024 Q5(b)

(b) A train departs from Connolly Station and accelerates uniformly from rest for  $t_1$  seconds until it reaches a speed of v. The train maintains this speed for  $t_2$  seconds, where  $t_2=2t_1$ . The train then decelerates to rest at Pearse Station in a time of  $t_3$  seconds.

The total time taken for the journey is  $T = t_1 + t_2 + t_3$ .

- (i) Show that  $T = \frac{2d t_2 v}{v}$ , where d is the distance between Connolly Station and Pearse Station.
- (ii) If the average speed for the entire journey is  $\frac{2v}{3}$ , show that  $T=6t_1$ .

#### Past Exam Questions:

2010: (a)(i) $1 m/s^2$ (ii) $1.17 m/s^2$ (b) $30$	00 m	2011: (a) $\frac{7}{8}$ s (b) (ii) t (iii) $\frac{31}{12}$ t		
2015: (a) 57 m (b)(i) 3.5 s (ii) 48.86 m	2016	b: (a)(ii) 5 m/s (iii) 107.5 m	2018: (b)(ii) $d = \frac{3u^2}{2a}$	
2024: Q1(c) $5 \pm \sqrt{2} s$				