

1) Indices

a) The Laws of Indices:

1) $a^p \times a^q = a^{p+q}$ e.g. $4^4 \times 4^3 = 4^7$

2) $\frac{a^p}{a^q} = a^{p-q}$ e.g. $\frac{5^3}{5^2} = 5^{3-2} = 5^1$

3) $(a^p)^q = a^{pq}$ e.g. $(5^2)^3 = 5^6$

4) $a^0 = 1$ e.g. $7^0 = 1$ or $(0.5)^0 = 1$

5) $a^{-p} = \frac{1}{a^p}$ e.g. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

6) $(ab)^p = a^p b^p$ e.g. $(3x)^2 = 3^2 x^2 = 9x^2$

7) $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$ e.g. $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

8) $a^{\frac{1}{2}} = \sqrt{a}$ e.g. $9^{\frac{1}{2}} = \sqrt{9} = 3$

9) $a^{\frac{1}{3}} = \sqrt[3]{a}$ e.g. $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

See Tables
pg 21

b) Solving equations with indices:

Steps:

1. Try and spot which powers you're dealing with, using the table below e.g. if you see a 9 and an 27 in the question, it would be powers of 3
2. Tidy up both sides of the equation into a single power using the laws of indices above. e.g. $5^x = 5^y$
3. If the bases are the same on both sides, you can now let the powers be equal to each other. i.e. $x = y$
4. Solve the simple equation to find your solution.

Example: Solve $3^x = 27\sqrt{3}$

$$3^x = 3^3 \cdot 3^{\frac{1}{2}} \dots \text{using Law 8 above on the } \sqrt{3}$$

$$3^x = 3^{3 + \frac{1}{2}} \dots \text{using Law 1}$$

$$3^x = 3^{7/2} \dots \text{tidying up the power into a single fraction}$$

$$\Rightarrow x = 7/2 \dots \text{as the bases are equal}$$

c) Table of Powers:

Note: It can be familiar to be able to recognise some of the more common powers. A table of them is shown below.

x	x ¹	x ²	x ³	x ⁴	x ⁵	x ⁶	x ⁷	x ⁸
2	2	4	8	16	32	64	128	256
3	3	9	27	81	243			
4	4	16	64	256				
5	5	25	125	625				
6	6	36	216					
7	7	49	343					
8	8	64	512					
9	9	81	729					
10	10	100	1000					

2) Surds:

Notes:

- A **surd** is a number in the form $\sqrt{\quad}$ that **can't be written** as a **rational** number i.e. in the form $\frac{a}{b}$
E.g. $\sqrt{2}$ and $\sqrt{3}$ are both surds but $\sqrt{9}$ is not as it can be written as $\frac{3}{1}$
- We can add/subtract similar surds together
E.g. i) $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$
ii) $4\sqrt{3} + 2\sqrt{2}$ we can't add these together as the $\sqrt{\quad}$ parts are different

Reducing Surds:

- We can use the rule $\sqrt{ab} = \sqrt{a}\sqrt{b}$ to reduce larger surds into a simpler form:

Example: Simplify $\sqrt{50} + \sqrt{32}$

- We use $50 = 25 \times 2$ rather than 10×5 as 25 is a square number)

$$\begin{aligned} \sqrt{50} + \sqrt{32} &= \sqrt{25}\sqrt{2} + \sqrt{16}\sqrt{2} \\ &= 5\sqrt{2} + 4\sqrt{2} \\ &= 9\sqrt{2} \end{aligned}$$