Topic 8: Calculus & Differential Equations

1) Integration:

a) Basic Rules for Integrating:

$$\int x^n . dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \frac{1}{x} . dx = \ln x + c$$

See Tables pg26

$$\int \frac{1}{ax+b} . dx = \frac{1}{a} \ln(ax+b) + c$$

Not in Tables

Definite Integrals:

$$\int_a^b f(x) . dx = F(b) - F(a)$$

$F(x)$ is the integral of f(x)

b) Integrating trig/exponentials:

$f(x)$	$\int f(x)dx$
e^x	e^x
e^{ax}	$\frac{1}{a}e^{ax}$
a^x	$\frac{a^x}{\ln a}$
$\cos x$	$\sin x$
$\sin x$	$-\cos x$
$\tan x$	$\ln\lvert\sec x\rvert$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a}$
$\frac{1}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$

See Tables pg 26

$$\int sin\ ax\ dx = -\frac{1}{a} cos\ ax + c$$

$$\int cos\ ax\ dx = \frac{1}{a} sin\ ax + c$$

NOT in Tables

c) Integration by Parts:

$$\int u . dv = uv - \int v . du$$

See Tables pg 26

Steps:

1. To select the correct u and dv we use the INLATE rule:
   Inverse Trig, Logs, Algebra, Trig, Exponents
2. The function that appears EARLIER from INLATE is u and let dv be the rest.
3. Differentiate and Integrate to get du and v.
4. Sub into rule above.
5. Might need more than one iteration to get solution.

d) Integration by Substitution:

Steps:

1. Let u = part of the function to be integrated to simplify the integration.
2. Find dx in terms of du.
3. Integrate simplified function.
4. Can change the limits of integration, or replace u with the function from step 1 and use the original limits.

2) Rates of Change/Differentiating Vectors:

1) Distance/Speed/Acceleration:

Distance s → Velocity v → Acceleration a

2) Differentiating Vectors:

- When differentiating vectors, we can differentiate the $\vec{i}$ and $\vec{j}$ components separately.

3) Work by Variable Force:

a) Work done by a Variable Force: **Steps:** 1. Get Force in terms of displacement x. 2. Use rule below to calculate work done between two distances a and b. $W = \int_a^b F(x)\,.\,dx$	

4) Differential Equations:

a) Type 1: 1st Order with general solutions **Steps:** 1. Multiply both sides by dx to eliminate fractions. 2. Gather all terms with a 'y' to one side, and the 'x' terms to the other. 3. Integrate both sides. 4. Get y on its own.	**b) Type 2: 1st Order with definite values** **Steps:** 1. Same first three steps as Type 1. 2. Use information given to evaluate the constant of integration. 3. Fill in the constant of integration and get y on its own.
c) Type 2: 2nd Order Separable **Steps:** 1. Let some variable be $\frac{dy}{dx}$ e.g. $\frac{dy}{dx} = v$ 2. Rewrite the given equation using v. 3. Proceed as in Type 1 and solve for v. 4. Work back using the substitution from step 1 and solve for y.	**d) Proportionality:** ➢ If two quantities P and Q are proportional to each other, we can say that P = kQ.

4) General Tips for the Exam:

- Do know your integration methods well
- Do separate the two sides carefully.
- Do put in a scale on all force diagrams.
- Do be careful with signs in the force equation.
- Do choose intelligently between the two possible expressions for acceleration.
- Don't confuse power and force.
- Don't put in the initial speed as a force.