## Topic 5: Functions/Graphs

### 1) The Basics:

**a) Terminology:**

- **Domain** = the values that are put **into** a function.
- **Range** = the values that come **out** of a function.
- **Codomain** = the values that **could come out** of a function.

**b) Notation:**

The different ways functions are written are:

- $f(x) = x^2 + 3x$
- $f: x \rightarrow x^2 + 3x$
- $y = x^2 + 3x$

**c) Evaluating Functions:**

**Example:** If $f(x) = 2x^2 + 3$, find $f(3)$ and $f(-1)$.

$f(3) = 2(3)^2 + 3 = 21$

$f(-1) = 2(-1)^2 + 3 = 5$

**d) Finding Inputs of Functions:**

**Example:** If $f(x) = 5x - 3$, find the value of $x$ for which $f(x) = 12$.

$f(x) = 12$

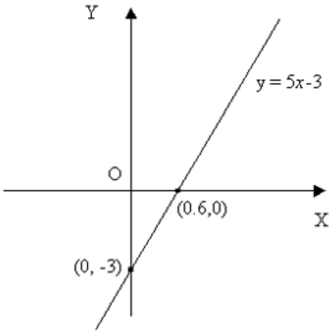
$\Rightarrow 5x - 3 = 12$

$\Rightarrow 5x = 15$

$\Rightarrow x = 3$

### 2) Types of Graphs:

**a) Linear: $y = ax + b$**

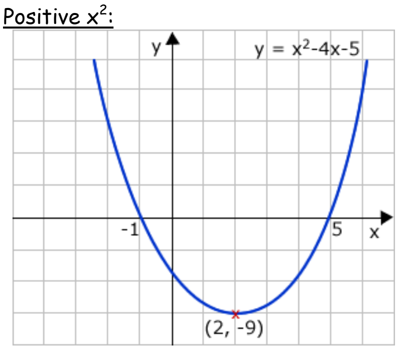



**Notes:**

- Graph above is for function of the form $y = ax + b$
- If 'a' is positive, the line increases from left to right but if 'a' is negative, the line decreases from left to right
- Function of the form $y = ax$ would be a line through the origin 'O'
- The root is where the graph crosses the x-axis….in the graph above, the root is 0.6.

**b) Quadratic: $y = ax^2 + bx + c$**

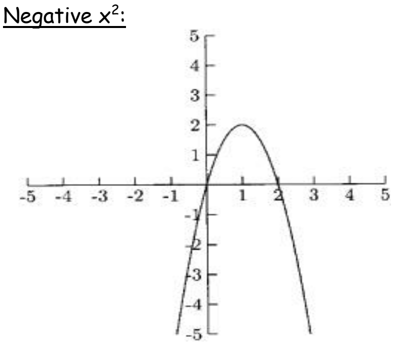
Positive $x^2$:




**Notes:**

- Graph above is for function of the form $y = ax^2 + bx + c$, where 'a' is a positive number
- Roots are where the graph crosses the x-axis…..in the graph above, the roots are -1 and 5
- The minimum point is the lowest point on the graph….in the graph above the minimum point is (2,-9).

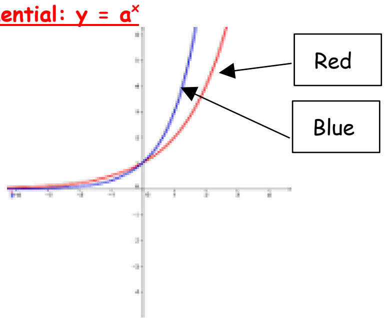
Negative $x^2$:



**Notes:**

- Graph above is for function of the form $y = ax^2 + bx + c$, where 'a' is a negative number
- Roots are where the graph crosses the x-axis…..in the graph above, the roots are 0 and 2
- The maximum point is the highest point on the graph…..in the graph above the maximum point is (1, 2)

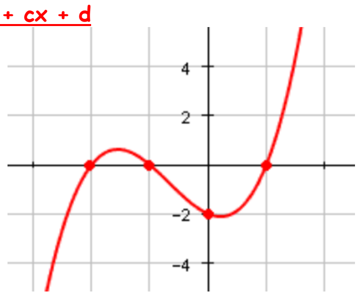
**c) Exponential: $y = a^x$**




**Notes:**

- Red line in the graph above is graph of $y = 2^x$ and blue line is graph of $y = 3^x$
- All graphs of the form $a^x$ pass through the point (0,1)
- Note that $y = 3^x$ rises at a steeper rate after passing through the point (0,1)
- Graphs of the form $a2^x$ would look similar to the graph of $2^x$ but would be shifted on the left

**d) Cubic: $y = ax^3 + bx^2 + cx + d$**



**Notes:**

- Graph above is for function of the form $y = ax^3 + bx^2 + cx + d$, where 'a' is a positive number
- If 'a' was negative the S shape would be the other way around i.e. coming in from the top left and leaving in the bottom right of the graph above
- Roots are where the graph crosses the x-axis, so cubic graphs have three roots
- The local minimum point is the point at the base of the trough in the graph….in the graph above the local minimum point is (0.4, -2)
- The local maximum is the peak of the hill in the graph…..in the graph above the local maximum point is (-1.5, 0.8)

### 3) Drawing/Interpreting Graphs:

**a) Drawing Graphs:**

- Just fill in the values from the domain and use calculator.

**Example:** Draw the graph of $x^2 - 3x - 4$, in the domain $-2 \leq x \leq 1$

$f(x) = x^2 - 3x - 4$

$f(-2) = (-2)^2 - 3(-2) - 4 = 6$ (-2, 6)
$f(-1) = (-1)^2 - 3(-1) - 4 = 0$ (-1, 0)
$f(0) = (0)^2 - 3(0) - 4 = -4$ (0, -4)
$f(1) = (1)^2 - 3(1) - 4 = -6$ (1, -6)
$f(2) = (2)^2 - 3(2) - 4 = -6$ (2, -6)
$f(3) = (3)^2 - 3(3) - 4 = -4$ (3, -4)

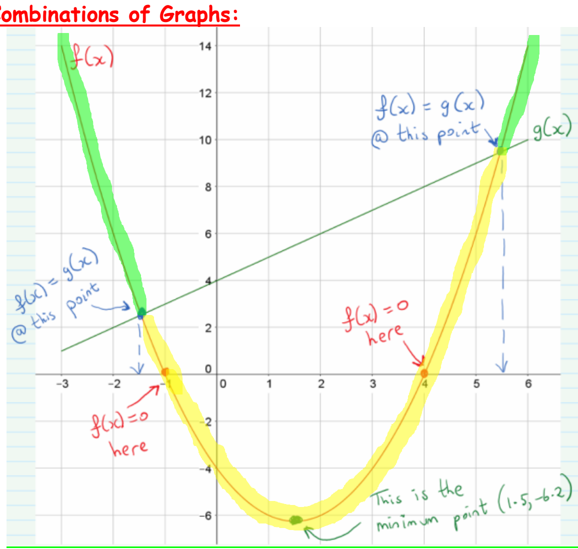
Can plot these on graph paper. Should know shape of graph from Section 2.

**b) Interpreting Graphs:**

**Tip:**
Use ruler and dotted lines when working out values from a graph

- To find f(2) or f(-1) from graph, for example: come up from x = 2 or x = -1 until you hit the graph and then go across to y value
- To find f(x) = 3 or f(x) = -2 from graph: draw a line through y = 3 or y = -2, and then come up/down to x-axis from the point(s) where the line crosses the graph
- Roots are where graph crosses x-axis i.e. f(x) = 0
- Axis of symmetry is the line that cuts the graph into 2. Only arises in U or ∩ shape.

**c) Combinations of Graphs:**




- f(x) = g(x) is where the two functions intersect
- Yellow Highlighted section is where f(x) < g(x)
- Green highlighted section is where f(x) > g(x)

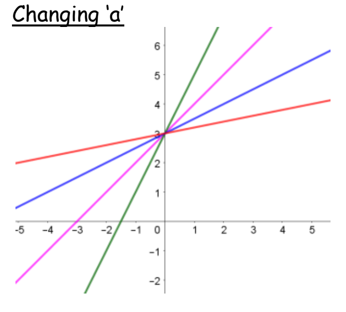
### 4) Graph Transformations:

**a) Linear Graphs (y = ax + b):**

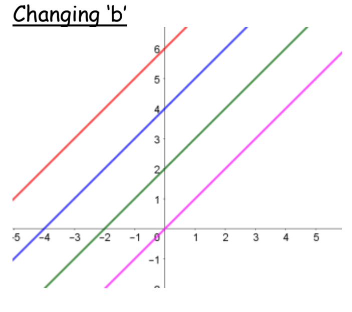
**Notes:**

- Changing the **'b'** changes the **y-intercept** to whatever the value of b is.
- Changing the **'a'** changes the **slope** of the graph to whatever the value of a is.

Changing 'a'



Changing 'b'

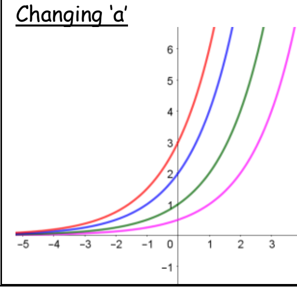


**b) Exponential Graphs ($y = ak^x + b$):**

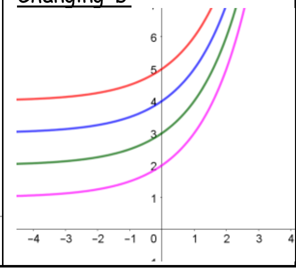
**Notes:**

- Changing the **'a'** in $y = ak^x$ changes the **y-intercept**. In a graph of the form $y = ak^x$, the graph crosses the y-axis at (0, a).
- Changing the **'b'** in $y = ak^x + b$ **shifts** the whole graph **up or down** crosses the y-axis at (0, b + 1). If b is positive it moves up and if it's negative it moves down.

Changing 'a'



Changing 'b'

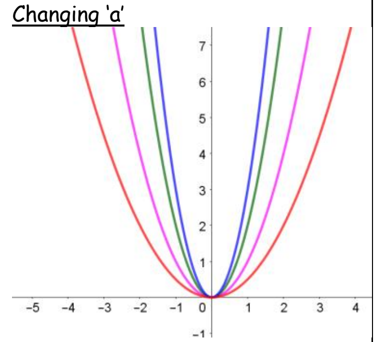


**c) Quadratic Graphs ($y = ax^2 + bx + c$):**

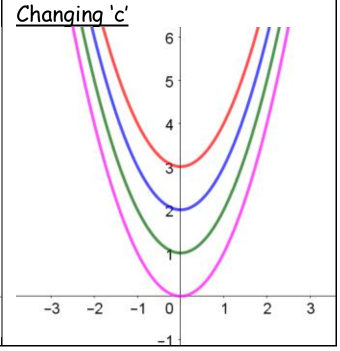
**Notes:**

- Changing the **'a' narrows** the **U shape**. The bigger the value of a the narrower it gets.
- Changing the **'c' shifts** the whole graph **up or down** depending on the value of c. If c is positive it moves up and if it's negative it moves down.

Changing 'a'



Changing 'c'

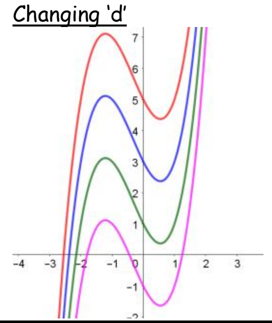


**d) Cubic Graphs ($y = ax^3 + bx^2 + cx + d$):**

**Notes:**

- Changing the **'d' shifts** the whole graph **up or down** depending on the value of d. If d is positive it moves up and if it's negative it moves down.
- If we **multiply** the entire function **by a constant** (e.g. 2) then the **max** and **min points** will be **twice** as high and low.

Changing 'd'



Multiplying by a constant