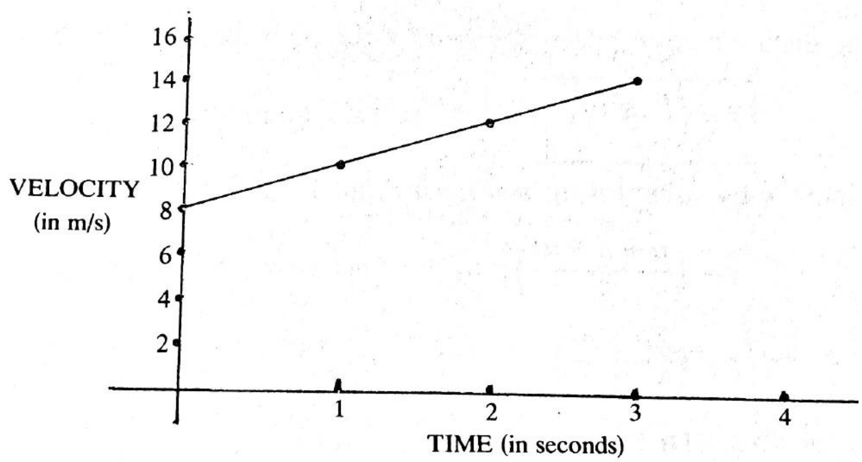
- Chapter 2: Uniform Acceleration
- Topic 8: Equations of Motion

- Time-Velocity Graphs:
  - Imagine a vehicle is travelling at 8m/s and then it's speed increases to 10m/s in 1 second.
  - After 2 seconds, it has increased to 12m/s and has increased to 14m/s after another second.
  - We can see that the vehicle is increasing its speed by 2m/s every second or it is accelerating at 2 metres per second per second i.e. 2m/s^2.
  - If we looked at a graph of the vehicle's speed versus time, it might look something like the graph below:



  - The graph is a straight line as the vehicle is accelerating at a steady rate of 2m/s^2.
  - We say the vehicle is accelerating uniformly.

- Equations of Motion:
  - You learned about speed, velocity and acceleration in Junior Science. We'll begin by reminding ourselves what each of them mean.
  - We already looked at the difference between Speed and Velocity in the last chapter.
  - Acceleration is the change in speed of a body, in a unit of time. The units we measure it in are usually m/s^2.
  - One formula we used for calculating it in Junior Science was:

$$Acceleration = \frac{Final\ Speed - Initial\ Speed}{Time\ Taken}$$

  - We now assign letters to each of the quantities in the formula, as follows:

    u = Initial Speed v = Final Speed t = time taken a = acceleration
  - This gives us the formula:

$$a = \frac{v-u}{t}$$

- If we multiply both sides of equation 1 by t, and then write 'v' in terms of the other variables we get:

$$v = u + at$$ Equation 1

- In general, the distance travelled 's' by a body is found by multiplying the average speed by the time taken. ("Dad's Silly Triangle):

$$s = \frac{(u+v)}{2}t$$ Equation 2

- We can now substitute our expression for 'v' from equation 2 into equation 3:

$$s = \frac{(u + u + at)}{2}t$$
$$s = \frac{(2u + at)}{2}t$$
$$s = \frac{2ut + at^2}{2}$$

$$s = ut + \frac{1}{2}at^2$$ Equation 3

- If we rearrange equation 1 and get 't' on its own we get:

$$t = \frac{v - u}{a}$$

- We now substitute this expression for 't' into equation 3 to get:

$$s = \frac{(u + v)}{2}t$$
$$s = \frac{(u + v)}{2}\frac{(v - u)}{a}$$
$$s = \frac{v^2 - u^2}{2a}$$
$$2as = v^2 - u^2$$

=> $$v^2 = u^2 + 2as$$ Equation 4

- In summary, we have now derived 4 very important equations that we can use to solve some problems.

$$v = u + at$$
$$s = \frac{(u+v)}{2}t$$
$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

See Tables Book pg 50

u = initial velocity
v = final velocity
a = acceleration
t = time taken
s = distance

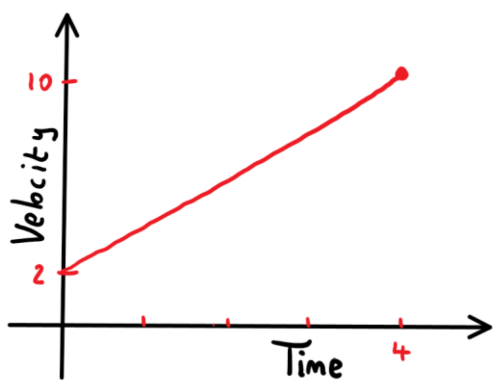
- If we are given any 3 of the variables u, v, a, t or s, we can find the other 2.

Note on Acceleration:

- If the value for 'a' is positive then the body is accelerating. If the value for 'a' is negative, it means the body is decelerating.
- For example, if 'a' is $-3m/s^2$, we say the acceleration is $-3m/s^2$ or the deceleration is $3m/s^2$.

- Example 1: Pg 31 Ex 2A Q2
  A cyclist speeds up uniformly from 2m/s to 10m/s over a 4 second period of time. Find the distance travelled.

  Solution:
  - The time-velocity graph for the motion is shown below:



- We have been given u = 2m/s, v = 10m/s and t = 4 secs. We can use fill these values into equation 2 above and solve for 's':

$$s = \frac{(u+v)}{2}t$$
$$s = \frac{(2+10)}{2}(4)$$
$$= 24m$$

Show them an alternative method by finding a first using v = u + at, and then using $s = ut + ½at^2$

Then ask them to calculate the area under the curve – nice link to tomorrow's lesson.

- Example 2: A truck is slowing down from 50m/s to rest. In doing so, it covers a distance of 150m. Find:

  i) the time taken

  ii) the deceleration

  iii) the distance the car would take to stop from 50m/s, if it's deceleration was doubled

Solution:

i) In this example, we know u = 50m/s, v = 0m/s and s = 150m, so we can use equation 2 to find the time taken:

$$s = \frac{(u + v)}{2} t$$

$$150 = \frac{(50 + 0)}{2} t$$

$$300 = 50t$$

$$t = \frac{300}{50} = 6\text{secs}$$

ii) We can now use equation 1 to find 'a':

v = u + at

0 = 50 + a(6)

-50 = 6a

$a = \frac{-50}{6} = -8.33m/s^2$

- If the acceleration is $-8.33m/s^2$, we say the deceleration is $8.33m/s^2$.

iii) If the deceleration was doubled, it would be -8.33 x 2 = $-16.66m/s^2$. We can now find the distance 's', using equation 4:

$v^2 = u^2 + 2as$

$(0)^2 = (50)^2 + 2(-16.66)s$

0 = 2500 – 33.33s

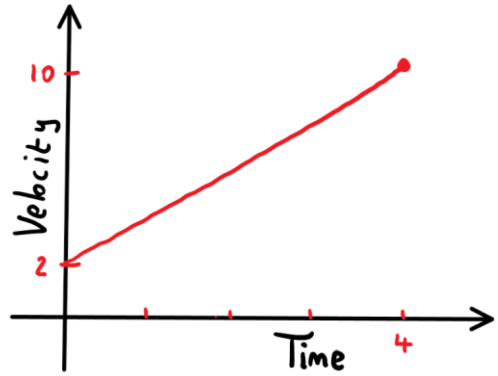
33.33s = 2500

$s = \frac{2500}{33.33} = 75m$

Classwork Questions: pg 31/32 Ex 2A Qs 3/5/7/9/11/13 and then try 18(c)/20

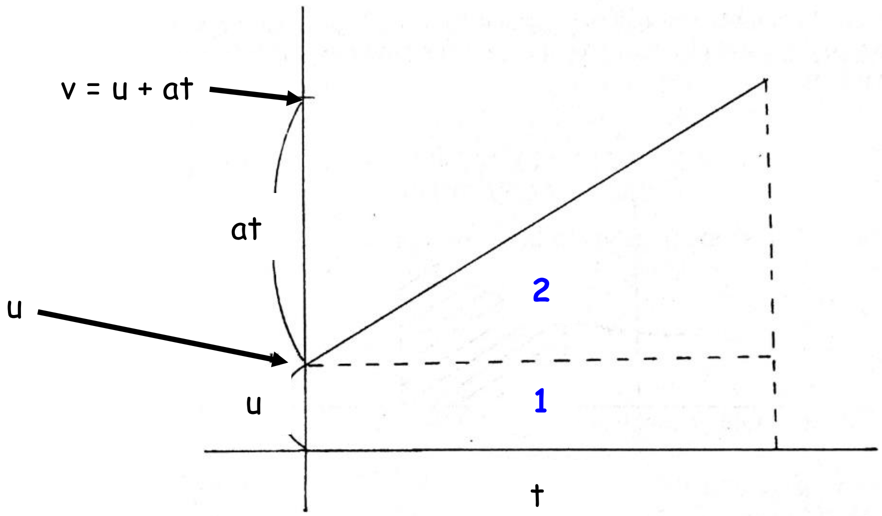
- Topic 9: Interpreting Time-Velocity Graphs

- We will start by returning to our initial example from yesterday.



Remind how we'd calculated area under the curve to get 40m. Want to prove to ye today that it holds in all cases.

- Since we know that v = u + at, we can redraw the diagram above to cover any situation:



- If we calculate the area under the graph in the diagram above, we get:

  Area Under Graph = Area Rectangle 1 + Area Triangle 2
  $= ut + \frac{1}{2}(at)(t)$
  $= ut + \frac{1}{2}at^2$

- We came across this expression before in equation 3 and it represents the distance travelled.
- So, in general:

The area under a time-velocity graph represents the total distance travelled.

- If the graph isn't linear, we need to use a topic you will come across in 6th year Maths known as Integration, to find the area under the graph.

- Example 1: Pg 34 Ex 2B Q3

  A car accelerates at 5 m/s2 from rest to a speed of 50 m/s. It then travels at a steady speed of 50 m/s for 25s and finally decelerates to rest with a deceleration of 10 m/s^2.

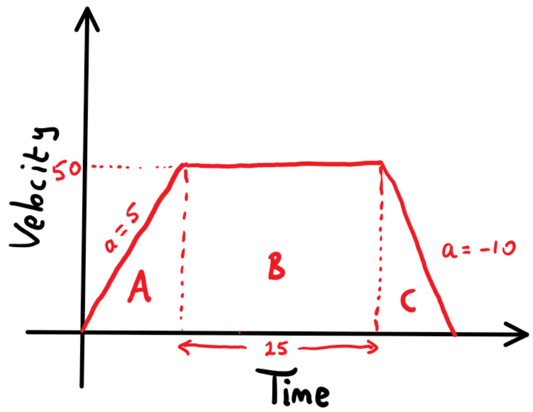
  Find:

  i) the total distance travelled

  ii) the average speed

  Solution:

  - The time-velocity graph for this situation is shown below:



i)

- To find the total distance travelled, we need to find the area under the graph.
- Before we can do that, we need to find the times for sections A and C:

Section A	Section C
$u = 0, v = 50, a = 5, t = ?$ $\Rightarrow v = u + at$ $\Rightarrow 50 = 0 + 5t$ $\Rightarrow t = \frac{50}{5} = 10$ secs	$u = 50, v = 0, a = -10, t = ?$ $\Rightarrow v = u + at$ $\Rightarrow 0 = 50 - 10t$ $\Rightarrow t = \frac{50}{10} = 5$ secs

- So, the total distance travelled will be:

Total Distance Travelled = Area A + Area B + Area C

= $\frac{1}{2}$(10)50) + (25)(50) + $\frac{1}{2}$(5)(50)

= 250 + 1250 + 125

= 1625 m

ii) The average speed can be found by dividing the total distance travelled by the total time taken:

$$\text{Average Speed} = \frac{1625}{10 + 25 + 5} = 40.625 \text{ m/s}$$

Day 1: Classwork Questions: pg 34/35 Ex 2B Qs 2/4/5/6 and then try Q7/10

Day 2: Classwork Questions: pg 36 Ex 2C Qs 2/3/4/6 → Let them try these and if they run into trouble, do Pg 36 Q1 as an example.

➢ Topic 10: Successive Posts Problems

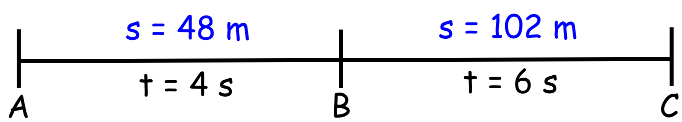
- We will now look at some more complicated examples.

• **Example:** Pg 37 Ex 2D Q1

A car moving with constant acceleration passes three posts, A, B and C, on a straight road. The distance from A to B is 48 m and from B to C is 102 m. The car takes 4 s to go from A to B and 6 s to go from B to C. Find the acceleration of the car.

Solution:

- It can be a good idea to draw a sketch showing what information we are given in each section:



- We will let the initial velocity or the car, as it passes A be 'u' and the acceleration be 'a'.
- We will look at the first section first:

$u = u$, $a = a$, $t = 4$, $s = 48$

$\Rightarrow s = ut + \frac{1}{2}at^2$

$48 = (u)(4) + \frac{1}{2}(a)(4)^2$

$48 = 4u + 8a$

$12 = u + 2a$……………..Equation X

- As we are using 'u' to represent the initial velocity as the train passes A, we have to look at sections 1 and 2 together rather than section 2 on its own:

$u = u$, $a = a$, $t = 4 + 6 = 10$, $s = 48 + 102 = 150$

$\Rightarrow s = ut + \frac{1}{2}at^2$

$150 = (u)(10) + \frac{1}{2}(a)(10)^2$

$150 = 10u + 50a$

$15 = u + 5a$……………..Equation Y

- We now solve equations X and Y to find 'u' and 'a':

X: $u + 2a = 12$

Y: $u + 5a = 15$

X: $u + 2a = 12$

-Y: $-u - 5a = -15$

$-3a = -3$

$\Rightarrow a = 1\ m/s^2$

Things to add on to example:

1) Show how we would find u if we needed it. u = 10 m/s
2) How do you handle if you're asked about another section of the journey after C?
3) Show alternative/quicker method of doing using average speed.

Classwork Questions: pg 37 Ex 2D Qs 2/3/5/6

- Topic 11: Problems with just Acceleration and Deceleration

- Example: Pg 39 Ex 2E Q5

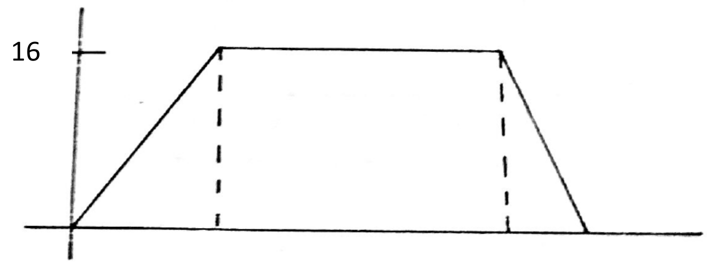
A car can accelerate with acceleration 1 m/s^2 and decelerate with deceleration 2 m/s^2. Find the least possible time it takes to cover a distance of 300m, from rest to rest,

i) subject to a speed limit of 16 m/s

ii) subject to no speed limit.

Solution:

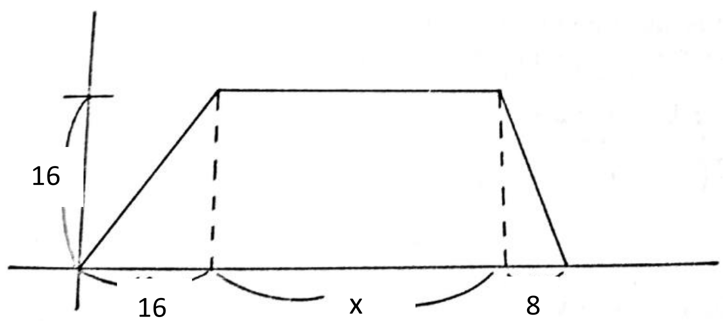
i) A time-velocity graph of the situation looks like:



- For the first and last parts of the journey, we can use equation 1 to find the times taken:

Accelerating Part	Decelerating Part
v = u + at 16 = 0 + (1)(t) 16 = t t = 16 secs	v = u + at 0 = 16 + (-2)(t) -16 = -2t t = 8 secs

- So, our time-velocity graph becomes:



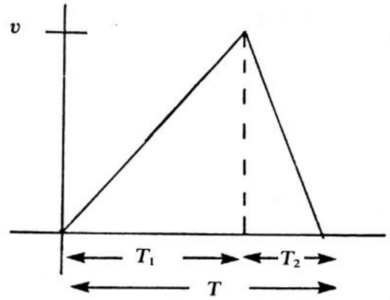
- We know the total distance travelled is 300m, so the area under the graph must be equal to 300m:

$$\text{Area under graph} = \tfrac{1}{2}(16)(16) + (x)(16) + \tfrac{1}{2}(8)(16)$$
$$300 = 128 + 16x + 64$$
$$300 = 192 + 16x$$
$$300 - 192 = 16x$$
$$108 = 16x$$
$$\Rightarrow x = \frac{108}{16} = 6.75\text{secs}$$

- We can now calculate the total time taken: 16 + 6.75 + 6 = 30.75 s

ii) There is no speed limit in this case, so the car accelerates to its maximum speed and then immediately decelerates again to rest.

- We will let the maximum speed be 'v' so a time-velocity graph looks like:



- We now look at both parts of the graph separately:

Accelerating part	Decelerating part
u = 0, v = v, t = T_1, a = 1 So, using equation 1 gives: v = u + at v = 0 + (1)(T_1) v = (1)T_1 => $T_1 = v$	u = v, v = 0, t = T_2, a = -2 So, using equation 1 gives: v = u + at 0 = v + (-2)(T_2) -v = (-2)T_2 => $T_2 = \frac{v}{2}$

- So, the total time taken is $T_1 + T_2$, which is :

$$= v + \frac{v}{2}$$
$$= \frac{2v}{2} + \frac{v}{2}$$
$$= \frac{3v}{2}$$

- Now, we can find the area under the curve, in terms of v:

$$\text{Area} = \frac{1}{2}\left(\frac{3v}{2}\right)(v) = 300$$
$$\Rightarrow \frac{3v^2}{4} = 300$$
$$\Rightarrow 3v^2 = 4(300)$$
$$\Rightarrow 3v^2 = 1200$$
$$\Rightarrow v^2 = \frac{1200}{3} = 400$$
$$\Rightarrow v = \sqrt{400} = 20 \text{ m/s}$$

- We can now find the total time taken using our expressions for T_1 and T_2 above:

Total time taken = $T_1 + T_2$
$$= v + \frac{v}{2}$$
$$= 20 + \frac{20}{2} = 30\ secs$$

Classwork Questions: pg 39 Ex 2E Qs 2/3/4/7

- ## Topic 12: Overtaking Problems

- **Example:** Pg 41 Ex 2F Q1

A car starts from rest at a point P and moves with acceleration $4m/s^2$. As it starts, another car passes P moving in the same direction at a uniform speed of 20m/s. Find at what time overtaking will occur and how far from P.

Draw Velocity-Time Graph of situation to show them.

Solution:

- In this type of question, the key to solving it is realising that at the instant the cars overtake each other at time T, the distances travelled by both from P are the same.
- Let's look at the two cars separately and see what we know about each:

Car A	Car B
$u_1 = 0$ m/s, $a_1 = 4$ m/s^2, $t_1 = T$, $s_1 = ??$ So, using equation 3 gives: $s_1 = ut + \frac{1}{2}at^2$ $s_1 = (0)(T) + \frac{1}{2}(4)(T)^2$ $s_1 = 2T^2$	$u_2 = 20$ m/s, $a_2 = 0$ m/s^2, $t_2 = T$, $s_2 = ??$ So, using equation 3 gives: $s_2 = ut + \frac{1}{2}at^2$ $s_2 = (20)(T) + \frac{1}{2}(0)(T)^2$ $s_2 = 20T$

- When the cars overtake we know the two distances are equal ($s_1 = s_2$) so:

$20T = 2T^2$

$10T = T^2$

$T^2 - 10T = 0$

$T(T - 10) = 0$

$T = 0$ Or $T = 10$

This is when the two cars were together at P initially.

- So, the car will overtake the other car after 10 seconds.
- We can now calculate the distance travelled using our expression for s_1 or s_2 from above:

$s_1 = 20T$

$= 20(10)$

$= 200$m

Show how to find Greatest Distance between them:

V = 0 + 4T V = 4T	V = 20 + 0T V = 20
Equating Speeds: 4T = 20 => T = 5	
Distance A = $2(5)^2$ = 50	Distance B = 20(5) = 100
=> Greatest Distance = 100 – 50 = 50m	

Classwork Questions: pg 41/42 Ex 2F Qs 2/3/4/6/7 and then try Qs 9/10

Topic 13: Motion under gravity

- You might recall learning about gravity in Junior Science.
- You learned about calculating mass and weight and when calculating weight, we used a figure of $10m/s^2$ for gravity.
- In reality, the figure is approximately $9.8m/s^2$, and all things accelerate at $9.8m/s^2$, when they are falling.
- This is another real-life example of uniform acceleration, so the equations of motion we developed earlier can be used here again.
- In general:

$$Falling => Acc = +g$$
$$Thrown\ Upwards => Acc = -g$$

- **Example:** Pg 43 Ex 2G Q1

  A stone is thrown vertically upwards from ground level with an initial speed of 35 m/s.

  i) After how long will the stone hit the ground?

  ii) Find the greatest height reached.

  Solution:

  i) $u = 35, a = -9.8, s = 0, t = ??$

  - Using equation 3:

  $s = ut + \frac{1}{2}at^2$

  $0 = (35)t + \frac{1}{2}(-9.8)t^2$

  $4.9t^2 - 35t = 0$

  $t(4.9t - 35) = 0$

  $=> t = 0 \text{ or } 4.9t - 35 = 0$

  $=> t = \frac{35}{4.9} = \frac{50}{7} = 7.1\ s$

  ii) At the greatest height the speed of the stone will be 0, so:

  $u = 35, a = -9.8, v = 0, s = ??$

  - Using equation 4:

  $v^2 = u^2 + 2as$

  $(0)^2 = (35)^2 + 2(-9.8)s$

  $0 = 1225 - 19.6s$

  $19.6s = 1225$

  $s = \frac{1225}{19.6} = 62.5m$

Ask them to calculate time to rise to max height and time to come back down from max height – note that they're the same.

Classwork Questions: pg 43 Ex 2G Qs 2/3/4/6/7

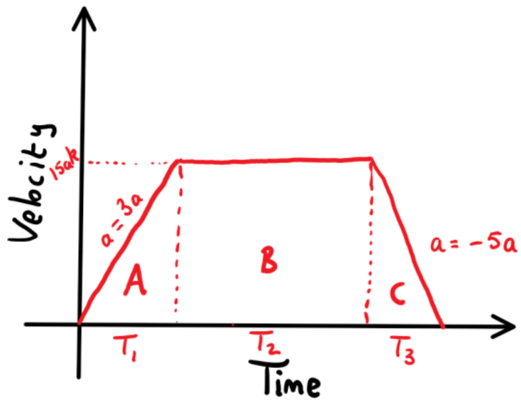
- Topic 14: More difficult problems

- Example: Pg 44 Ex 2H Q2(i)

  A car can move with acceleration 3a and deceleration 5a. Find, in terms of k, the time taken to cover a distance of $90ak^2$ from rest to rest subject to a speed limit of 15ak.

  Solution:

  - Firstly, we will draw a velocity-time graph of the situation and fill in as much information as we have:



  - When we learned about these types of problems before, the first thing we did was to try and get expressions for the times for sections A and C, so:

Section A	Section C
u = 0 , v = 15ak, t = T_1, a = 3a So, using equation 1 gives: v = u + at 15ak = 0 + (3a)(T_1) 15ak = 3aT_1 => T_1 = 5k (dividing both sides by 3a)	u = 15ak, v = 0, t = T_3, a = -5a So, using equation 1 gives: v = u + at 0 = 15ak + (-5a)(T_3) 5aT_3 = 15ak => T_3 = 3k (dividing both sides by 5a)

  - We know the total distance travelled is $90ak^2$, so:

$$\text{Total Area} = \text{Area A} + \text{Area B} + \text{Area C}$$
$$90ak^2 = \tfrac{1}{2}(5k)(15ak) + (T_2)(15ak) + \tfrac{1}{2}(3k)(15ak)$$
$$90ak^2 = \tfrac{75}{2}ak^2 + 15akT_2 + \tfrac{45}{2}ak^2$$

  - Multiplying through by 2 and dividing through by ak gives:

$$180k = 75k + 30T_2 + 45k$$
$$\Rightarrow 30T_2 = 180k - 75k - 45k$$
$$\Rightarrow 30T_2 = 60k$$
$$\Rightarrow T_2 = 2k$$

  - So, now we can finish and calculate the total time taken in terms of k:

$$\text{Total Time} = T_1 + T_2 + T_3$$
$$= 5k + 2k + 3k = 10k$$

Classwork Questions: pg 44/35 Ex 2H Qs 1/4/6(i)/2(ii)/3

Revision Questions and Test