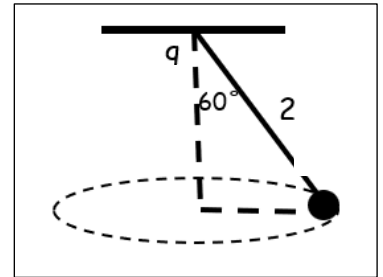
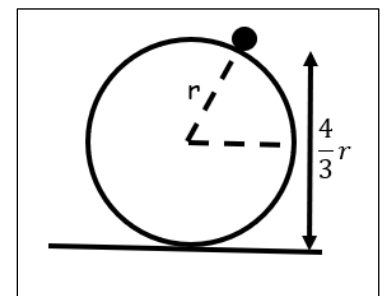


Horizontal Circular Motion Problems:

- Q1. A smooth particle of mass 3 kg is attached to the end of a light inextensible string of length 60 cm . The particle moves in a horizontal circle with constant angular velocity of 3 rad/s on a smooth horizontal surface.
- Find the centripetal acceleration of the particle.
 - Find the tension in the string.
- Q2. A smooth particle of mass 5 kg is attached to a light inextensible string of length 80 cm . The particle describes a horizontal circle on a smooth horizontal table with a constant linear velocity of $v\text{ m/s}$. If the maximum tension in the string is 62.5 N , find the maximum value of v possible without the string breaking.
- Q3. A particle is placed on a playground merry-go-round, at a distance of 80 cm from the centre. The coefficient of friction between the particle and the merry-go-round is 0.3 . The speed of rotation of the merry-go-round is slowly increased from zero. At what angular velocity will the particle begin to slip?
- Q4. A particle of mass 4 kg is attached via a light inextensible string of length 2 m to a fixed point q on a ceiling, as shown in the diagram on the right. The particle moves in a horizontal circle of radius r , with the centre of the circle being directly below q . The string makes an angle of 60° with the vertical.
- Find the value of r .
 - Find the tension in the string.
 - Find the angular velocity of the particle.

Vertical Circular Motion Problems:

- Q5. A smooth circular hoop, of radius r , is fixed in a vertical plane. A small ring, of mass m , is threaded on the hoop and projected from the lowest point on the wire with speed u .
- If $u = \sqrt{3gr}$ find the maximum height reached by the particle.
 - What is the height of the particle above the lowest point when its speed $v = \sqrt{\frac{3}{2}gr}$.
 - Find the minimum value of u for the particle to just reach the highest point on the wire.
- Q6. A body of mass m is hanging from a light inextensible string of length 2 m . The body is at rest with the string vertical. The body is then given a horizontal velocity of 8 m/s . Find the angle the string makes with the upward vertical at the instant where the string goes slack.
- Q7. A particle can move on the outside surface of a smooth sphere of radius r . The sphere is fixed to a horizontal table. The particle is released from rest at a point on the outer surface of the sphere, which is $\frac{4}{3}r$ above the table, as shown in the diagram on the right. Find, in terms of r , the height of the particle above the table when it leaves the surface of the sphere.

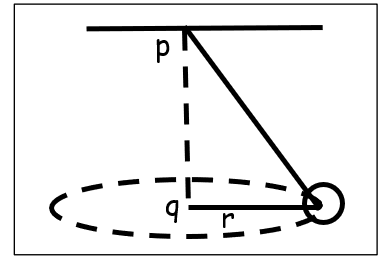


Hooke's Law Problems:

- Q8.** A body of mass 0.1 kg is attached to the midpoint of an elastic string pq . The string pq has elastic constant 3.5 N/m and natural length 1.5 m . The string is stretched between two fixed points, with p being 2.5 m vertically above q . Find the distance from p to the particle when the particle is in equilibrium.
- Q9.** An elastic string of natural length 1.5 m and elastic constant $3g \text{ N/m}$ is suspended vertically with a body of mass 3 kg hanging from its end.
- Find the length of the string when its hanging in equilibrium.
 - The body is pulled 0.5 m below the equilibrium position and released. Find its initial upward acceleration after being released.
- Q10.** An elastic string of natural length 1 m and elastic constant $k \text{ N/m}$ is suspended from a ceiling with a body of mass 2 kg hanging from its end. The body is pulled to the side, and it is given a velocity of $v \text{ m/s}$ so that the system rotates as a conical pendulum. The body makes a circle of radius 0.5 m , and the vertical distance between the particle and ceiling is 1.2 m .
- Show that: (i) $k = \frac{65g}{9}$ (ii) $v = \sqrt{\frac{5g}{24}}$

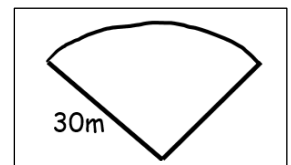
Mixture:

- Q11.** The ends p and q of a light inelastic string, of length 1 m , are attached to two fixed points as shown in the diagram on the right. Point p is 50 cm above q and a ring of mass 1 kg is attached to the string at the point shown. The ring then moves in a horizontal circle with centre of q and radius r .



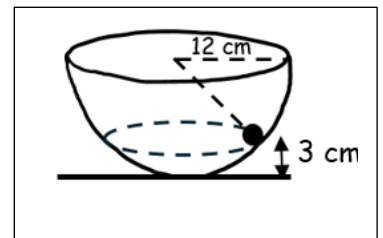
- Calculate the radius r of the circle.
- Find the tension in the string.
- Find the angular velocity of the ring.

- Q12.** A humpback bridge is in the shape of a circular arc of radius 30 m as shown in the diagram. What is the maximum speed the car can travel over the top of the bridge, without leaving the road?



- Q13.** A marble is moving on the inside surface of a smooth sphere. The radius of the sphere is 3 m . The marble is describing a horizontal circle of radius $\sqrt{2} \text{ m}$. Calculate the angular velocity of the particle.

- Q14.** A smooth particle of mass 5 kg describes a horizontal circle on the inside surface of a smooth hemisphere of radius 12 cm , as shown in the diagram on the right. The radius of the circle of motion is r and the centre of the circle of rotation is 3 cm above the horizontal surface the sphere is on.



- Find the value of r .
- Find the normal reaction between the particle and the hemisphere's surface.
- Show that the angular velocity of the particle is $\frac{\sqrt{g}}{3} \text{ rad/s}$.

Final challenges:

- Q15.** An elastic string of natural length l m and elastic constant k N/m is stretched between two points which are $4l$ m apart on the same horizontal ceiling. A particle of weight W N is attached to the midpoint of the string and hangs in equilibrium with the two strings both inclined at 30° to the horizontal. Find k in terms of W and l .
- Q16.** A small ring is threaded on a smooth circular hoop, of radius r , which is fixed in a vertical plane. The ring is projected with speed u from the highest point of the hoop p . At the lowest point of the hoop q it collides with a second ring of equal mass, and the two rings coalesce. What is the smallest value of u , in terms of r , for which the two rings will reach point q in the subsequent motion?
- Q17.** A light elastic string of natural length 1.2 m extends by 50 cm when a small mass m is suspended from the string hanging vertically in equilibrium. If the mass is then made to describe a horizontal circle with angular velocity ω rad/s, find an expression for the extension of the string in terms of ω .
- Q18.** The ends of a light string of length $2l$ are attached to two points, where one point is $2a$ vertically above the other. A smooth ring of weight W is threaded onto the string. Show that if the ring revolved in a horizontal circle level with the lower point of attachment of the string, then its velocity is $\sqrt{\frac{(l^2 - a^2)g}{a}}$ and the tension in the string is $\frac{W(l^2 + a^2)}{2la}$.
- Q19.** A body is held at a point P on the surface of a smooth fixed sphere of radius r and centre O , where OP makes an angle of 60° with the upward vertical. The body is released and rolls from rest down the outside of the sphere. Find the vertical height above O of the point where the particle loses contact with the sphere and find also the horizontal distance from O of the particle when it is level with O .

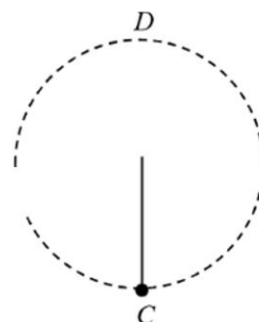
Answers:

Q1. (i) 5.4 m/s^2 (ii) 16.2 N	Q2. $\sqrt{10} \text{ m/s}$ or 3.16 m/s	Q3. $\frac{7\sqrt{30}}{20} \text{ rads/s}$ or 1.92 rads/s
Q4. $\sqrt{g} \text{ rads/s}$ or $\frac{7\sqrt{5}}{5} \text{ rads/s}$ or 3.13 rads/s	Q5. (i) $\frac{3}{2}r$ (ii) $\frac{3}{4}r$ (iii) $\sqrt{5gr} \text{ m/s}$	Q6. 65.9°
Q7. $\frac{11r}{9} \text{ m}$	Q8. 1.39 m	Q9. (i) 2.5 m (ii) 4.9 m/s^2
Q11. (i) 0.375 m (ii) $\frac{5g}{4} \text{ N}$ or 12.25 N (iii) $\frac{28\sqrt{15}}{15} \text{ rads/s}$ or 7.23 rads/s	Q12. $7\sqrt{6} \text{ m/s}$ or 17.15 m/s	
Q13. 1.92 rads/s	Q14. (i) $3\sqrt{7} \text{ m/s}$ or 7.94 cm (ii) $\frac{20g}{3} \text{ N}$ or 65.33 N (iii) $\frac{\sqrt{g}}{3} \text{ rads/s}$	
Q15. $\frac{2\sqrt{3}W}{l(8-\sqrt{3})}$ or $\frac{0.55W}{l}$	Q16. $2gr\sqrt{3} \text{ m/s}$	Q17. $\frac{1.2\omega^2}{2g-\omega^2}$ or $\frac{1.2\omega^2}{19.6-\omega^2}$
Q19. $\frac{r}{3}$ and $1.02r$		

Past Exam Questions:

2016 Q6

6. (a) A small particle hanging on the end of a light inextensible string 2 m long is projected horizontally from the point C .
- (i) Calculate the least speed of projection needed to ensure that the particle reaches the point D which is vertically above C .
- (ii) If the speed of projection is 7 m s^{-1} find the angle that the string makes with the vertical when it goes slack.

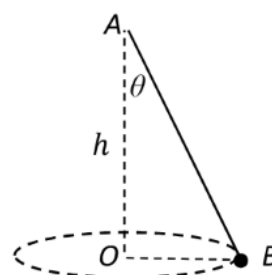


2018 Q6

- (b) A particle P is attached to one end of a light inextensible string of length d . The other end of the string is attached to a fixed point. The particle is hanging freely at rest, with the string vertical, when it is projected horizontally with speed u . The particle moves in a complete vertical circle.
- (i) Show that $u \geq \sqrt{5gd}$.
- As P moves in the circle the least tension in the string is T_1 and the greatest tension is kT_1 .
- (ii) Given that $u = \sqrt{6gd}$, find the value of k .

2019 Q6

6. (a) One end A of a light elastic string is attached to a fixed point. The other end, B , of the string is attached to a particle of mass m . The particle moves on a smooth horizontal table in a circle with centre O , where O is vertically below A and $|AO| = h$. The string makes an angle θ with the downward vertical and B moves with constant angular speed ω about OA .



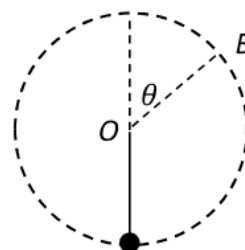
- (i) Show that $\omega^2 \leq \frac{g}{h}$.

The elastic string has natural length h and elastic constant $\frac{2mg}{h}$.

- (ii) Given that $\omega^2 = \frac{2g}{5h}$, find the value of θ .

2020 Q6

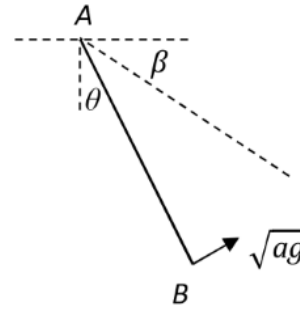
- (b) A particle P is attached to one end of a light inextensible string of length d . The other end of the string is attached to a fixed point O . The particle is hanging freely at rest, with the string vertical, when it is projected horizontally with speed $\sqrt{3gd}$. The particle moves in a vertical circle. The string becomes slack when P is at the point B . OB makes an angle θ with the upward vertical.



- (i) Show that $\cos \theta = \frac{1}{3}$.
- (ii) In terms of d , find the greatest height of P above B in the subsequent motion.

2017 Q6

- (b) One end A of a light inextensible string of length $3a$ is attached to a fixed point. A particle of mass m is attached to the other end B of the string. The string makes an angle θ with the vertical.



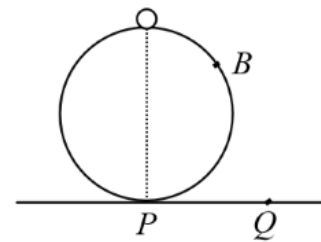
The particle is held in equilibrium with the string taut and $\cos \theta = \frac{2}{3}$. The particle is then projected with speed $\sqrt{ag}$, in the direction perpendicular to AB , as shown in the diagram. In the subsequent motion the string remains taut.

When AB makes an angle β below the horizontal, the speed of the particle is v and the tension in the string is T .

- (i) Show that $v^2 = 3ag(2 \sin \beta - 1)$.
- (ii) Find the minimum value and the maximum value of T .

2010 Q6

6. (a) A particle of mass m kg lies on the top of a smooth sphere of radius 2 m. The sphere is fixed on a horizontal table at P .

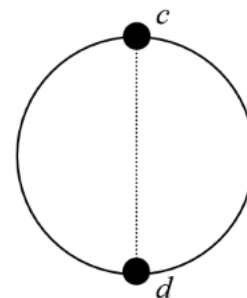


The particle is slightly displaced and slides down the sphere. The particle leaves the sphere at B and strikes the table at Q .

- Find
- (i) the speed of the particle at B
 - (ii) the speed of the particle on striking the table at Q .

2007 Q6

- (b) A bead slides on a smooth fixed circular hoop, of radius r , in a vertical plane. The bead is projected with speed $\sqrt{10gr}$ from the highest point c . It impinges upon and coalesces with another bead of equal mass at d . cd is the vertical diameter of the hoop.



Show that the combined mass will not reach the point c in the subsequent motion.

Past Exam Questions:

2016: $u = 7\sqrt{2}$ and $\alpha = 80.41^\circ$	2018: $k = 7$	2019: 36.87°	2020: (ii) $4d/27$
2017: $T_{\min} = 0.5mg, T_{\max} = 2mg$		2010: (i) $\sqrt{\frac{4g}{3}}$ (ii) $\sqrt{8g}$	