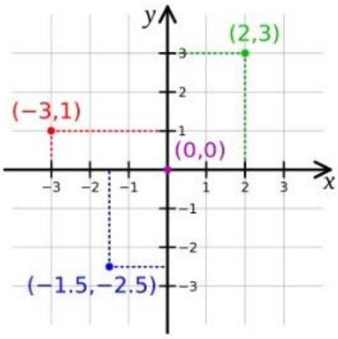
Topic 11: Coordinate Geometry (The Line and The Circle)

1) The Basics:

a) Cartesian Plane/Coordinates:

Notes:

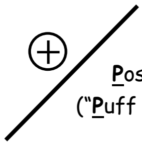
- Coordinates must be listed in brackets with a comma in between the two numbers
- We always list the X value first and the Y value second…see examples in diagram above.
- The point (0,0), shown in purple, is also called the **Origin**.
- The X and Y axes divides the plane up into 4 **quadrants**
  - Quadrant 1 is top right of the plane and they are numbered in an anti-clockwise direction



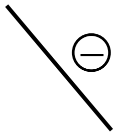
c) Slope:

Notes:

- Slope is a measure of the steepness of a line.
- Slopes can be negative or positive:

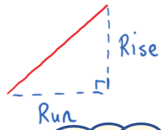


Positive Slope
("Puff Puff Positive")

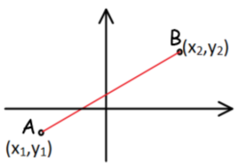


Negative Slope
("Nice Negative")

- There are three different ways we can find it:

Formula when we know **2 points**:	When given **diagram**:	When given the **equation of the line** in the form ax + by + c = 0
Slope |AB| $\frac{y_2 - y_1}{x_2 - x_1}$	 Slope = $\frac{RISE}{RUN}$	$\frac{-x\ number}{y\ number}$
Tables pg 18	**Not** in Tables	**Not** in Tables

b) Distance/Midpoint Formula:



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Distance |AB|

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of AB

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

d) Equation of a line:

Notes:

- A unique licence plate that identifies a particular line.
- To use the formula, we have to know:
  - **A point on the line**
  - The **slope** of the line (See section above)
- Once we know the two things above we use the formula:

$$y - y_1 = m(x - x_1)$$

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- The equation of a line can also be given in the form:

$$y = mx + c$$

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where **'m' = the slope and 'c' = the y-intercept** (where the line crosses the y-axis)

Example: A line with equation y = 3x – 5 has a slope of 3 and crosses the y-axis at the point (0, -5).

e) Intersecting Lines:

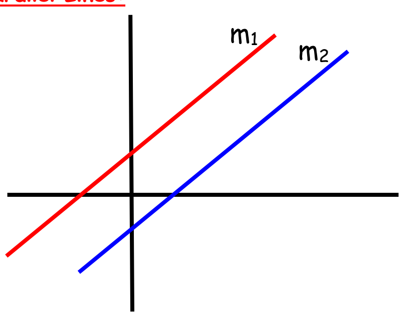
We can find where two lines meet by solving the equations simultaneously. See Algebra – Section 5a

f) Graphing/Sketching Lines:

Easiest method: Find where the line crosses the x-axis (y = 0) and the y-axis (x = 0)

2) Parallel/Perpendicular Lines:

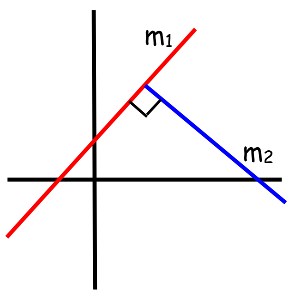
a) Parallel Lines:



Not in Tables

$$m_1 = m_2$$

b) Perpendicular Lines:



Not in Tables

$$m_1 \times m_2 = -1$$

N.B. If you have one slope, you can **'Flip & Change'** to get the other one

e.g. if $m_1 = 2/5$

=> $m_2 = -5/2$

3) Area of a Triangle:

a) Triangle with one point at (0,0):

Note:

- To find the area of a triangle using the formula below, one of the points must be (0, 0).

Area = $\frac{1}{2}|x_1y_2 - x_2y_1|$

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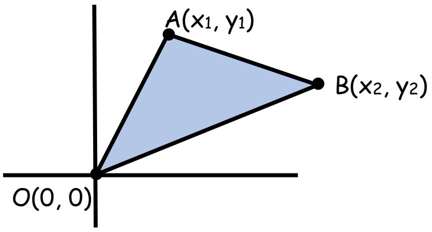


Example 1: Find the area of the triangle with coordinates (0,0), (4,-1) and (5,-3).

Area = $\frac{1}{2}|x_1y_2 - x_2y_1|$ (x_1, y_1) = (4, -1) (x_2, y_2) = (5, -3)

Area = $\frac{1}{2}|(4)(-3) - (5)(-1)|$

Area = $\frac{1}{2}|-12 + 5|$

Area = $\frac{1}{2}|-7|$ (taking the positive value of what's in the | |)

Area = $\frac{1}{2}(7) = 3.5$units²

b) Triangle with no points at (0,0):

Note:

- If none of the points are (0, 0), you have to move one point to (0, 0) and move the other points under the same translation.

Example 2: Find the area of the triangle with coordinates (3,-1), (5,2) and (-2,-3).

- Choose one point e.g. (3, -1) and move it to (0, 0) first and then move the other points by the same:
  - (3, -1) --------> (0, 0)   (take 3 from x, add 1 to y)
  - (5, 2)---------> (2, 3)   (take 3 from x, add 1 to y)
  - (-2, -3)-------> (-5, -2)   (take 3 from x, add 1 to y)
- Now proceed as Example 1 with the three new points:

Area = $\frac{1}{2}|x_1y_2 - x_2y_1|$ (x_1, y_1) = (2, 3) (x_2, y_2) = (-5, -2)

Area = $\frac{1}{2}|(2)(-2) - (3)(-5)|$

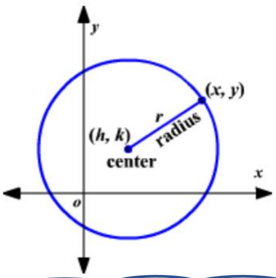
Area = $\frac{1}{2}|-4 + 15|$

Area = $\frac{1}{2}|11|$

Area = $\frac{1}{2}(11) = 5.5$units²

4) Types of Circles:

a) Circle with Centre other than (0,0)



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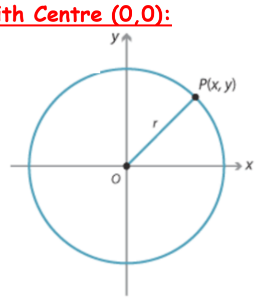
Equation: $(x-h)^2 + (y-k)^2 = r^2$

b) Circle with Centre (0,0):



Equation: $x^2 + y^2 = r^2$

Not in Tables but can find by subbing in (0,0) for (h,k) in other formula on the left.

5) Points Inside, On or Outside a Circle:

Method 1:

Steps:

1. Write down the radius and centre of the circle.
2. Calculate distance from the point to the centre.
3. Compare distance to radius:
   - If Distance < Radius => Point is Inside
   - If Distance > Radius => Point is Outside
   - If Distance = Radius => Point is On Circle

Method 2:

Steps:

1. Fill in point into equation of the circle.
2. Compare left hand side to right hand side.
   - If LHS < RHS => Point is Inside
   - If LHS > RHS => Point is Outside
   - If LHS = RHS => Point is On Circle

Example: Is the point (6, -2) in, on or outside the circle $(x-2)^2 + (y+3)^2 = 25$

Method 1:	Method 2:
R = $\sqrt{25}$ = 5 Centre = (2, -3) Dist from (2,-3) to (6,-2): $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ $\sqrt{(6-2)^2 + (-2+3)^2}$ $\sqrt{17} = 4.12$ $4.12 < 5$ => INSIDE circle	$(x-2)^2 + (y+3)^2 = 25$ $(6-2)^2 + (-2+3)^2 = 25$ $(4)^2 + (1)^2 = 25$ $17 < 25$ => INSIDE circle

6) Intersection of a Line and a Circle:

- Need to be able to find the points of intersection of a line and a circle.
- See Algebra Topic Section 5b