Worked Solutions

Q1. $a = 12 \quad S_\infty = 36$

$$S_\infty = \frac{a}{1-r}$$

$$\Rightarrow \frac{12}{1-r} = \frac{36}{1}$$

$$36(1-r) = 12$$

$$36 - 36r = 12$$

$$\frac{36r}{36} = \frac{24}{36}$$

$$\boxed{r = \frac{2}{3}}$$

Q2. $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$1, 2, 3, 4 \ldots\ldots 200$

$a = 1 \quad d = 1 \quad n = 200$

$$S_{200} = \frac{200}{2}\{2(1) + (200-1)(1)\}$$

$$= 100\{2 + 199\}$$

$$= 100(201)$$

$$= \boxed{20100}$$

Q3. $x = 0.2525\ldots\ldots$

$100x = 25.2525\ldots\ldots$

$$\Rightarrow \frac{99x}{99} = \frac{25}{99}$$

$$\Rightarrow x = \boxed{\frac{25}{99}}$$

Q4. $T_n = 3n + 2$

$a = 1^{st}$ term $= T_1$

$T_1 = 3(1) + 2$

$= 5$

$T_2 = 3(2) + 2$

$= 8$

$\Rightarrow d = 8 - 5 = 3$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{n}{2}\{2(5) + (n-1)(3)\}$$

$$= \frac{n}{2}\{10 + 3n - 3\}$$

$$= \boxed{\frac{n}{2}(3n+7)}$$

or $\boxed{\dfrac{3n^2 + 7n}{2}}$

Q5. $S_n = 3n^2 - 4n$

i) 1^{st} term $T_1 = S_1 = 3(1)^2 - 4(1)$

$= \boxed{-1}$

ii) $S_2 = 3(2)^2 - 4(2) = 4$

$\Rightarrow -1 + T_2 = 4$

$T_2 = 5$

Arithmetic $\Rightarrow d = 5 - (-1) = 6$

$\Rightarrow T_3 = 5 + 6 = 11$

$\Rightarrow T_2 + T_3 = 5 + 11 = \boxed{16}$

Q6. Let 3 nos: $x-d,\ x,\ x+d$

$$Sum = 27$$
$$\Rightarrow x - \cancel{d} + x + x + \cancel{d} = 27$$
$$\frac{3x}{3} = \frac{27}{3}$$
$$x = 9$$

$$Product = 704$$
$$\Rightarrow (x-d)(x)(x+d) = 704$$
$$(9-d)(9)(9+d) = 704$$
$$(81 - 9d)(9+d) = 704$$
$$729 + \cancel{81d} - \cancel{81d} - 9d^2 - 704 = 0$$
$$-9d^2 + 25 = 0$$
$$9d^2 - 25 = 0$$
$$(3d)^2 - (5)^2 = 0$$
$$(3d+5)(3d-5) = 0$$
$$3d+5 = 0 \quad \text{or} \quad 3d-5=0$$
$$\frac{3d}{3} = \frac{-5}{3} \qquad \frac{3d}{3} = \frac{5}{3}$$
$$d = \frac{-5}{3} \qquad d = \frac{5}{3}$$

$$\Rightarrow 9 - \frac{5}{3},\ 9,\ 9 + \frac{5}{3}$$
$$\boxed{\frac{22}{3},\ 9,\ \frac{32}{3}}$$

Q7. Geometric $\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_2}$

$$\Rightarrow \frac{x+1}{2x-4} = \frac{x-3}{x+1}$$
$$(2x-4)(x-3) = (x+1)(x+1)$$
$$2x^2 - 4x - 6x + 12 = x^2 + x + x + 1$$
$$x^2 - 12x + 11 = 0$$
$$(x-1)(x-11) = 0$$
$$x - 1 = 0 \quad \text{or} \quad x - 11 = 0$$
$$\boxed{x = 1} \qquad \boxed{x = 11}$$

Q8. i)
$$T_2 = a + (2-1)d = a + d$$
$$T_5 = a + (5-1)d = a + 4d$$
$$T_2 + T_5 = 18$$
$$a + d + a + 4d = 18$$
$$2a + 5d = 18 \ : \mathrm{I}$$

$$T_3 = a + (3-1)d = a + 2d$$
$$T_6 = a + (6-1)d = a + 5d$$
$$T_6 - T_3 = 9$$
$$a + 5d - (a + 2d) = 9$$
$$\cancel{a} + 5d - \cancel{a} - 2d = 9$$
$$\frac{3d}{3} = \frac{9}{3}$$
$$\boxed{d = 3}$$

Using I: $\Rightarrow 2a = 18 - 5(3)$
$$2a = 3$$
$$\boxed{a = \frac{3}{2}}$$

ii) $S_n = \frac{n}{2}\{2a + (n-1)d\}$
$$\frac{n}{2}\left(2\left(\tfrac{3}{2}\right) + (n-1)(3)\right) > 600$$
$$n(\cancel{3} + 3n - \cancel{3}) > 1200$$
$$3n^2 > 1200$$
$$n^2 > 400$$
$$n > \sqrt{400} = 20 \Rightarrow \boxed{n = 21}$$

Q9. i) $a = 2 \quad r = \frac{1}{3}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{2\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}}$$

$$= 3\left(1-\left(\frac{1}{3}\right)^n\right)$$

ii) $3\left(1-\left(\frac{1}{3}\right)^n\right) = \frac{242}{81}$

$$1-\left(\frac{1}{3}\right)^n = \frac{242}{243}$$

$$\left(\frac{1}{3}\right)^n = \frac{1}{243}$$

$$\log\left(\frac{1}{3}\right)^n = \log\left(\frac{1}{243}\right)$$

$$n.\log\left(\frac{1}{3}\right) = \log\left(\frac{1}{243}\right)$$

$$n = \frac{\log\left(\frac{1}{243}\right)}{\log\left(\frac{1}{3}\right)}$$

$$= \boxed{5}$$

Q10. $x = 0.636363\ldots.$

$100x = 63.636363$

$\Rightarrow \frac{99x}{99} = \frac{63}{99}$

$$x = \frac{63}{99} = \boxed{\frac{7}{11}}$$

Q11. Let 3 nos: $\frac{x}{r}, x, rx$

Sum = 62 :

$\Rightarrow \frac{x}{r} + x + rx = 62$

$x + rx + r^2x = 62r \quad : \mathrm{I}$

Product = 1000 :

$\Rightarrow \left(\frac{x}{r}\right)(x)(rx) = 1000$

$x^3 = 1000$

$x = \sqrt[3]{1000}$

$= 10$

$\Rightarrow$ Using I :

$10 + 10r + 10r^2 = 62r$

$10r^2 - 52r + 10 = 0$

$5r^2 - 26r + 5 = 0$

$(5r - 1)(r - 5) = 0$

$5r - 1 = 0 \qquad r - 5 = 0$

$5r = 1 \qquad r = 5$

$r = \frac{1}{5}$

$\Rightarrow$ 3 nos : $\frac{10}{5}, 10, 5(10)$

$\boxed{2, 10, 50}$

Q12. $S_\infty = \frac{a}{1-r}$

$a = 4 - 3x \qquad r = 4 - 3x$

$$\frac{4-3x}{1-(4-3x)} = \frac{2}{3}$$

$$\frac{4-3x}{1-4+3x} = \frac{2}{3}$$

$3(4-3x) = 2(3x-3)$

$12 - 9x = 6x - 6$

$18 = 15x$

$x = \frac{18}{15} = \boxed{\frac{6}{5}}$ or $\boxed{1.2}$

Q13. $T_n = S_n - S_{n-1}$

i) $3n^2 - 7n - [3(n-1)^2 - 7(n-1)]$

$3n^2 - 7n - [3(n^2 - 2n + 1) - 7n + 7]$

$3n^2 - 7n - 3n^2 + 6n - 3 + 7n - 7$

$= 6n - 10$

$T_n - T_{n-1}$ should be constant

$6n - 10 - (6(n-1) - 10)$

$6n - 10 - (6n - 6 - 10)$

$6n - 10 - 6n + 16$

$= 6 \Rightarrow$ Arithmetic

ii) Common difference $= \boxed{6}$

$a = 1^{st}$ term $= T_1$

$T_n = 6n - 10$

$T_1 = 6(1) - 10$

$= \boxed{-4}$

Q14. $T_5 = a + (5-1)d = a + 4d$

$T_2 = a + (2-1)d = a + d$

$T_5 = 2 \times T_2$

$a + 4d = 2(a + d)$

$a + 4d = 2a + 2d$

$a - 2d = 0 \Rightarrow a = 2d$

$T_5 - T_2 = 9$

$a + 4d - (a + d) = 9$

$3d = 9$

$d = 3$

$\Rightarrow a = 2(3) = 6$

$S_{10} = \frac{10}{2}\{2(6) + (10-1)(3)\}$

$= \boxed{195}$

Q15. $T_3 = a + (3-1)d = a + 2d$

$T_7 = a + (7-1)d = a + 6d$

$T_3 = 71$ and $T_7 = 55$

$\Rightarrow \cancel{a} + 2d = 71 \quad : \mathrm{I}$

$(-)\cancel{a} \overset{(-)}{+} 6d \overset{(-)}{=} 55$

$-4d = 16$

$\boxed{d = -4}$

Using I:

$a + 2(-4) = 71$

$a - 8 = 71$

$a = 71 + 8 = \boxed{79}$

Q16. i) Geo $\Rightarrow a = 3 \quad r = 2$

$T_n = a.r^{n-1}$

$= \boxed{3.(2^{n-1})}$

ii) $3(2^{n-1}) > 1000000$

$2^{n-1} > \frac{1000000}{3}$

$\log(2^{n-1}) > \log\left(\frac{1000000}{3}\right)$

$(n-1)\log 2 > \log\left(\frac{1000000}{3}\right)$

$n - 1 > \frac{\log\left(\frac{1000000}{3}\right)}{\log 2}$

$n - 1 > 18.3466$

$n > 19.3466$

$\boxed{n = 20}$

Q17. $S_\infty = \frac{a}{1-r} = \frac{9}{2}$

$T_2 = ar^{n-1}$
$= ar^{2-1}$
$= ar = -2$
$\Rightarrow a = \frac{-2}{r}$ ✱

$\frac{a}{1-r} = \frac{9}{2}$

$2a = 9(1-r)$

$2a = 9 - 9r$

Subbing in ✱:

$2\left(\frac{-2}{r}\right) = 9 - 9r$

$-4 = 9r - 9r^2$

$9r^2 - 9r - 4 = 0$

$(3r-4)(3r+1) = 0$

$3r - 4 = 0 \qquad 3r + 1 = 0$

$\frac{3r}{3} = \frac{4}{3} \qquad \frac{3r}{3} = \frac{-1}{3}$

$r = \frac{4}{3} \qquad r = -\frac{1}{3}$

S_∞ formula only valid if $-1 < r < 1$

$\Rightarrow \boxed{r = -\frac{1}{3}}$

Q18. $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$S_5 = \frac{5}{2}\{2a + (5-1)d\}$
$= \frac{5}{2}(2a + 4d)$
$= 5a + 10d = 50$
$\Rightarrow a + 2d = 10 \quad \text{I}$

$S_{10} = \frac{10}{2}\{2a + (10-1)d\}$
$= 5(2a + 9d)$
$= 10a + 45d$

Sum of 2nd 5 terms $= S_{10} - S_5$

$\Rightarrow 10a + 45d - (5a + 10d) = 125$

$10a + 45d - 5a - 10d = 125$

$5a + 35d = 125$

$a + 7d = 25 \quad \text{II}$

Solving I & II:

$\cancel{a} + 2d = 10$
$(-)\ \cancel{a} \overset{(-)}{+} 7d \overset{(-)}{=} 25$

$\frac{-5d}{-5} = \frac{-15}{-5}$

$d = 3$

Using I:

$a + 2d = 10$

$a + 2(3) = 10$

$a + 6 = 10$

$a = 10 - 6$

$\boxed{a = 4}$

Q19. $T_1 = 5$

$T_2 = 55 = 50 + 5$

$T_3 = 555 = 500 + 50 + 5$

$T_4 = 5555 = 5000 + 500 + 50 + 5$

Reversing T_4:

$$T_4 = \underbrace{5 + 50}_{\times 10} \underbrace{+ 500}_{\times 10} \underbrace{+ 5000}_{\times 10}$$

$\Rightarrow$ Geometric Series with

$a = 5 \quad r = 10$ and $n = 4$

$\Rightarrow T_n =$ Geo Series with

$a = 5 \quad r = 10$ and $n = n$

$$\Rightarrow S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{5(10^n - 1)}{10 - 1}$$

$$= \boxed{\frac{5}{9}(10^n - 1)}$$

Q20. $U_n = 2\left(-\frac{1}{2}\right)^n - 2$

i) $U_{n+1} = 2\left(-\frac{1}{2}\right)^{n+1} - 2$

$$= 2\left(-\frac{1}{2}\right)^n\left(-\frac{1}{2}\right)^1 - 2$$

$$= \boxed{-1\left(-\frac{1}{2}\right)^n - 2}$$

$U_{n+2} = 2\left(-\frac{1}{2}\right)^{n+2} - 2$

$$= 2\left(-\frac{1}{2}\right)^n\left(-\frac{1}{2}\right)^2 - 2$$

$$= \boxed{\frac{1}{2}\left(-\frac{1}{2}\right)^n - 2}$$

ii) $2U_{n+2} - U_{n+1} - U_n = 0$

$$2\left(\frac{1}{2}\left(-\frac{1}{2}\right)^n - 2\right) - \left(-1\left(-\frac{1}{2}\right)^n - 2\right) - \left(2\left(-\frac{1}{2}\right)^n - 2\right) = 0$$

$$1\left(-\frac{1}{2}\right)^n - 4 + 1\left(-\frac{1}{2}\right)^n + 2 - 2\left(-\frac{1}{2}\right)^n + 2 \stackrel{?}{=} 0$$

$$0\left(-\frac{1}{2}\right)^n + 0 \stackrel{?}{=} 0$$

$$0 = 0 \quad \checkmark$$

Q21. $T_2 = a.r^{2-1} = ar = 8$

$T_5 = ar^{5-1} = ar^4 = 27$

$ar = 8$

$\Rightarrow a = \frac{8}{r}$

$ar^4 = 27$

$\left(\frac{8}{r}\right)r^4 = 27$

$8r^3 = 27$

$r^3 = \frac{27}{8}$

$r = \sqrt[3]{\frac{27}{8}}$

$\boxed{r = \frac{3}{2}}$

$ar = 8$

$a\left(\frac{3}{2}\right) = 8$

$3a = 16$

$\boxed{a = \frac{16}{3}}$

Q22. $U_1 = 2 \qquad U_{n+1} = (-1)^n U_n + 3$

$\Rightarrow U_2 = (-1)^1(2) + 3 = \boxed{1}$

$U_3 = (-1)^2(1) + 3 = \boxed{4}$

$U_4 = (-1)^3(4) + 3 = \boxed{-1}$

$U_5 = (-1)^4(-1) + 3 = \boxed{2} = U_1$

$U_6 = U_2$

$U_7 = U_3$

$U_8 = U_4$

$U_9 = U_5$

$\Rightarrow U_{10} = U_6 = U_2 = \boxed{1}$

Q23. a, b, c are first 3 of an arithmetic

$\Rightarrow b - a = c - b$

$2b = a + c \qquad \text{I}$

a, c, b are first 3 of a geometric

$\Rightarrow \frac{c}{a} = \frac{b}{c}$

$\Rightarrow c^2 = ab \qquad \text{II}$

<u>Put I into II :</u>

$c^2 = ab$

$c^2 = a\left(\frac{a+c}{2}\right)$

$2c^2 = a(a+c)$

$2c^2 = a^2 + ac$

$2c^2 - ac - a^2 = 0$

$(2c + a)(c - a) = 0$

$2c + a = 0 \qquad c - a = 0$

$2c = -a \qquad \cancel{c = a}$

$\Rightarrow a = -2c \qquad$ 3 different nos.

Common ratio $= \frac{c}{a}$

$= \frac{c}{-2c}$

$= \boxed{-\frac{1}{2}}$

Q24. $T_1 = a \qquad T_3 = b$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$2d = T_3 - T_1$

$2d = b - a$

$d = \frac{b-a}{2}$

$\Rightarrow S_n = \frac{n}{2}\left\{2a + (n-1)\left(\frac{b-a}{2}\right)\right\}$

$S_4 = \frac{4}{2}\left\{2a + (4-1)\left(\frac{b-a}{2}\right)\right\}$

$= 2\left(2a + 3\left(\frac{b-a}{2}\right)\right)$

$= 4a + 3(b-a)$

$= 4a + 3b - 3a$

$= \boxed{a + 3b}$

$S_5 = \frac{5}{2}\left\{2a + 4\left(\frac{b-a}{2}\right)\right\}$

$= 5a + 5(b-a)$

$= 5b$

$S_7 = \frac{7}{2}\left\{2a + 6\left(\frac{b-a}{2}\right)\right\}$

$= 7a + \frac{21}{2}(b-a)$

$= 7a + \frac{21b}{2} - \frac{21a}{2}$

$= \frac{21b}{2} - \frac{7a}{2}$

Geo Series $\Rightarrow \frac{S_5}{S_4} = \frac{S_7}{S_5}$

$\Rightarrow (S_5)^2 = S_4 . S_7$

$\Rightarrow (5b)^2 = (a+3b)\left(\frac{21b}{2} - \frac{7a}{2}\right)$

$25b^2 = \cancel{\frac{21ab}{2}} - \frac{7a^2}{2} + \frac{63b^2}{2} - \cancel{\frac{21ab}{2}}$

$50b^2 = -7a^2 + 63b^2$

$7a^2 = 63b^2 - 50b^2$

$\boxed{7a^2 = 13b^2}$

Q25. $S_n = u_1 + u_2 + \cdots + u_n$

$\Rightarrow S_{n-1} = \boxed{u_1 + u_2 + \cdots + u_{n-1}}$

$\underline{S_n - S_{n-1}}$:

$$\cancel{u_1} + \cancel{u_2} + \cdots \cancel{u_{n-1}} + u_n$$

$$- \cancel{u_1} + \cancel{u_2} + \cdots + \cancel{u_{n-1}}$$

$$= \boxed{u_n}$$

Q26. Arithmetic

$$\Rightarrow \frac{1}{x} - \frac{1}{a} = \frac{1}{b} - \frac{1}{x}$$

$$\frac{1}{x} + \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

$$\frac{2}{x} = \frac{a+b}{ab}$$

Flip both sides:

$$\frac{x}{2} = \frac{ab}{a+b}$$

$$\Rightarrow \boxed{x = \frac{2ab}{a+b}}$$

Q27. Initial intensity = 2000

i) Mirror 1 $= \frac{3}{5}(2000) = 1200$

Mirror 2 $= \frac{3}{5}(1200) = 720$

Mirror 3 $= \frac{3}{5}(720) = 432$

$\Rightarrow \underset{0}{2000}, \underset{1}{1200}, \underset{2}{720}, \underset{3}{432} \ldots$

$a = 2000 \quad r = 0.6$

$T_{10} = 2000(0.6)^{10}$

$= \boxed{12}$

ii) $T_n = \boxed{2000(0.6)^n}$

iii) Intensity $= \frac{1}{10}$ of $2000 = 200$

$2000(0.6)^n = 200$

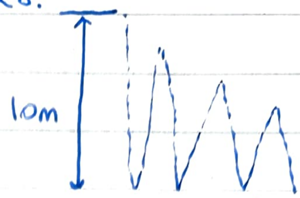
$(0.6)^n = \frac{1}{10}$

$\log(0.6)^n = \log\left(\frac{1}{10}\right)$

$n = \frac{\log(1/10)}{\log(0.6)} = 4.5$

$\Rightarrow \boxed{5^{th} \text{ mirror}}$

Q28.



i) Height = 0.6 (Prev height)

Height after 1 bounce = 6m

Height after 2 bounces = 3.6m

Height after 3 bounces = $\boxed{2.16m}$

ii) $10 + 2(6) + 2(3.6) + 2(2.16) + \ldots$

$10 + \underbrace{12 + 7.2 + 4.32 + \ldots}_{\text{Geo Ser: } a = 12 \quad r = 0.6}$

$10 + \frac{12(1-(0.6)^n)}{1-0.6}$

$10 + 30(1-(0.6)^n)$

$10 + 30 - 30(0.6)^n$

$S_n = 40 - 30(0.6)^n$

Before strikes for 5^{th} time

$\Rightarrow n = 4$

$S_4 = 40 - 30(0.6)^4$

$= \boxed{36.112m}$

iii) $10 + \underbrace{12 + 7.2 + 4.32 + \ldots}_{\text{Infinite Geo S}}$

$\Rightarrow 10 + \frac{12}{1-0.6} \quad \left(S_\infty = \frac{a}{1-r}\right)$

$= 10 + 30$

$= \boxed{40m}$

Q29. $x = 5.122222$

$\Rightarrow 10x = 51.22222$

$\Rightarrow 100x = 512.22222$

$\Rightarrow 90x = 461 \quad \Rightarrow x = \boxed{\frac{461}{90}}$