Topic 1: Arithmetic

1) The Basics:

a) Types of Numbers:

- Natural (N): Positive Whole Numbers: e.g. 1, 2, 3,
- Integers (Z): Positive and Negative Whole Numbers: e.g. -3, -2, -1, 0, 1, 2, 3,......
- Real (R): All numbers: e.g.s -3, -1.4, 0.2, 6, 7/2, $\sqrt{8}$
- Rational (Q): Numbers that <u>can</u> be written in the form $\frac{a}{b}$ e.g.s -5, 3, 1/2, -9/4....
- Irrational: Numbers that <u>cannot</u> be written in the form $\frac{a}{b}$ e.g.s $\sqrt{3}$, $\sqrt{2}$, π
- **Prime:** A natural number bigger than 1 with only itself and 1 as divisors. e.g.s 2, 3, 5, 7, 11, 13, 17.....
- Composite: A number that is not prime. e.g.s., 6, 9, 15, 20

b) Rounding:

Rounding to Decimal Places:

- To round to 2 decimal places, look at the 3rd number after the decimal point.
 - If it's 5 or more we round UP the 2nd number
 - If it's 4 or less we round DOWN the 2nd number
- Similar approach for rounding to other decimal places

Examples: i) 4.768 = 4.77 ii) 3.2745 = 3.27

- Rounding to Significant Figures:

 To round to 3 significant figures, we look at the 4th significant
 - If it's 5 or more we round UP the 3rd number and replace subsequent numbers with 0s
 - If it's 4 or less we round DOWN the $3^{\rm rd}$ number and replace subsequent numbers with 0s

Examples: i) 132,421 = 132,000 ii) 0.00472543 = 0.00473

c) Scientific Notation

Notes:

- > A number is in scientific notation if it's in the form $a \times 10^n$, where 'a' has to be between 1 and 10.
 - **Examples:** i) $3400 = 3.4 \times 10^3$ ii) $0.004 = 4 \times 10^{-3}$
- On a Casio calculator the button you will need to type in numbers in scientific notation is:



- \triangleright To type in 7 x 10⁴, press "7" and the button above and then "4"
- \succ To convert numbers into scientific notation on your calculator:
 - Type in the number and press = to enter it on the screen.
- Press "Shift" + "Mode" and select "Sci" from the menu. Then press "0".

d) Foreign Exchange

Steps:

- 1. Write the conversion with the currency you want on the right.
- 2. Get a 1 on the left-hand side, by dividing both sides.
- 3. Multiply both sides to get the value you want.

Example: If €1 = \$1.32, how many euro would you get for \$200? Step 1: Put euro on the right

Step 2: Get a 1 on the left-hand side

\$1 =
$$\epsilon \frac{1}{1.32}$$
 (dividing both sides by 1.32)

Step 3: Multiply both sides

\$200 =
$$\frac{1}{1.32}$$
 x 200 = €151.52

e) HCF/LCM using Prime Factors:

Notes:

- When asked to find the HCF and LCM of 2 numbers using prime factors, use your calculator.
- > Type in the number first and then press Shift + Button below.



- For HCF: pick the lowest power of each prime number.
- For LCM: pick the highest power of each prime number.

Example: Find the HCF and LCM of 60 and 80.

Using calculator:
$$60 = 5^1 \times 3^1 \times 2^2$$
 $80 = 2^4 \times 5^1$

HCF:

- Lowest Power of 2 is 2²
- Lowest Power of 3 is 3^1but 3^1 doesn't go into 80 so exclude
- Lowest Power of 5 is $5^1 = HCF$ of 60 and 80 is $2^2 \times 5^1 = 10$

LCM:

- Highest Power of 2 is 2⁴
- Highest Power of 3 is 31
- Highest Power of 5 is $5^1 = \lambda LCM$ of 60 and 80 is $2^4 \times 3^1 \times 5^1 = 240$

f) Speed, Distance and Time:

Notes

For all speed, distance and time calculations remember:
"Dads Silly Triangle"



If you want Distance, you cover the D in the triangle, so:

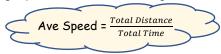
 $D = S \times T$ (Units are usually m or km)

If you want **Time**, you cover the **T** in the triangle, so: T = D / S (Units are usually secs or hrs)

> If you want **Speed**, you cover the **S** in the triangle, so:

S = D / T (Units might be m/s or km/h)

Average speed can be calculated using:



Careful with units of time. E.g. 1hr 45mins = 1.75hrs etc.

g) Ratio

Notes:

- Ratio shows how to break up a quantity proportionally.
- > When given a ratio, add the values in the ratio together to get the total number of parts the quantity is being broken into.
- Write down the fraction each person gets.

Example: Divide €200 between Alan and Brian in the ratio 3:2.

- 3:2 means there are 3+2=5 parts
- => Alan gets \(\frac{3}{2}\) and Brian gets \(\frac{2}{2}\) => Alan gets \(\frac{3}{2}\) of \(\inftige 200\) = \(\inftige 120\) and Brian gets \(\frac{2}{2}\) of \(\inftige 200\) = \(\inftige 80\)

2) Percentages/Profit/Loss/VAT:

a) Percentages:

To find the percentage of a number:

Example: Find 24% of 250. Method 1: Calculate $\frac{24}{100} x \frac{250}{1} = 60$ Method 2: Multiply 250 by 0.24 = 60

To find the total when given percentage:

Example: 25% of the marks in an exam are going for the practical part. If there are 50 marks for the practical, how many marks is the whole exam worth?

Steps:

1) Let % = value you're given

25% = 50

2) Find what 1% represents by dividing both sides

1% = $\frac{50}{25}$ = 2

3) Find 100% by multiplying by 100:

 $100\% = 2 \times 100 = 200$ marks

Note:

In this particular example, we could also have just multiplied 50 by 4, as 25% represents 1/4 of the total marks

% Profit or Mark-Up = $\frac{Profit}{Cost Price} \times 100 \%$ % Loss = $\frac{Loss}{Cost \, Price} \, x \, 100 \, \%$ % Discount = $\frac{Discount}{Cost\ Price}$ x 100 %

c) VAT:

VAT excluded:

Example: Bill comes to €120. Find final bill with 13.5% VAT.

VAT = 13.5% of 120 = 120 × 0.0135 = €16.20

=> Final Bill = €120 + €16.20 = €136.20

VAT included:

Example: Bill including VAT comes to €340.50. Find bill without

VAT, if VAT is 13.5%.

Bill + VAT = €340.50 => 113.5% = €340.50

=> 1% = €3 => 100% = €300

b) % Profit / Loss / Discount:

3) Income Tax:

a) Income Tax Terminology:

- Gross Income: total pay someone gets before any taxes or deductions are taken
- Net Income: Take home pay
- Standard Rate of Tax (usually about 20%)
- Gross Tax: Total tax owing to the government before credits
- Tax Credits: Money deducted from the gross tax
- Tax Payable: Tax that you actually pay

b) Answering Questions:

- The guestions are nearly always made up of 3 parts:
- Part 1: Calculation of Gross Tax
- Part 2: Calculation of Tax Payable using the equation Tax Paid = Gross Tax - Tax Credits
- Part 3: Working out Net Income by taking off all deductions including Tax Paid.

4) Compound Interest/Depreciation:

a) Terminology:

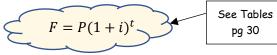
- Principal: Amount of money invested or borrowed
- Interest: Money added by the bank
- Rate: what percentage the interest is added at
- Amount or Final Value: The value of money at the end of the term it has been borrowed or invested for.

b) Answering Compound Interest Questions:

Method 1: Used if rates change from year to year or payments/withdrawals are being made between years

- Lay out Year 1, Year 2, Year 3 etc.
- Work out interest each year and add to Principal at start of

Method 2: Formula



where **F** is the Amount, **P** is the Principal, **i** is the Rate of Interest as a decimal (e.g. 3% = 0.03) and t is the time in years the money is invested/borrowed for.

5) Household Bills:

Notes:

- With utility bills (e.g.s. gas, electricity, water) there is usually a unit rate i.e. a charge per unit used
- To calculate the units used, subtract the previous units reading from the current units reading
- With many bills there is also a standing charge that has to be added on.
- VAT is also added to the bills.
- With Gas Bills, there is also a Carbon Tax that needs to be added on.

Example: Calculate the cost of electricity if the previous meter reading was 21310 and the current reading is 21836, with a standing charge of €21.60. The cost per unit is €0.15 and VAT of 13.5% is added on.

> Units used = Current Reading - Previous Reading = 21836 - 21310 = 526 units

Cost for electricity = 526 x €0.15 = €78.90

Standing Charge = €21.60

=> Total Before VAT = €78.90 + €21.60 = €100.50

VAT = 13.5% of €100.50 = €13.57

=> Final Bill = €100.50 + €13.57 = €114.07