Topic 1: Arithmetic

1) The Basics:

a) Types of Numbers:

- **Natural (N)**: Positive Whole Numbers: e.g. 1, 2, 3, ...
- **Integers (Z)**: Positive and Negative Whole Numbers: e.g. -3, -2, -1, 0, 1, 2, 3,.....
- **Real (R):** All numbers: e.g.s -3, -1.4, 0.2, 6, 7/2, $\sqrt{8}$ ... .
- **Rational (Q):** Numbers that **cannot** be written in the form $\frac{a}{b}$ e.g.s -5, 3, 1/2, -9/4.....
- **Irrational:** Numbers that **cannot** be written in the form $\frac{a}{b}$ e.g.s $\sqrt{3}$, $\sqrt{2}$, π ... ...
- **Prime:** A natural number bigger than 1 with only itself and 1 as divisors. e.g.s 2, 3, 5, 7, 11, 13, 17.....
- **Composite:** A number that is not prime. e.g.s., 6, 9, 15, 20

b) Rounding:

Rounding to Decimal Places:

- To round to 2 decimal places, look at the 3rd number after the decimal point.
  - If it's 5 or more we round UP the 2nd number
  - If it's 4 or less we round DOWN the 2nd number
- Similar approach for rounding to other decimal places

Examples: i) 4.768 = 4.77 ii) 3.2745 = 3.27

Rounding to Significant Figures:

- To round to 3 significant figures, we look at the 4th significant figure.
  - If it's 5 or more we round UP the 3rd number and replace subsequent numbers with 0s
  - If it's 4 or less we round DOWN the 3rd number and replace subsequent numbers with 0s

Examples: i) 132,421 = 132,000 ii) 0.00472543 = 0.00473

c) Scientific Notation

Notes:

- A number is in scientific notation if it's in the form **a x 10^n**, where 'a' has to be between 1 and 10.
  Examples: i) 3400 = 3.4 x 10^3 ii) 0.004 = 4 x 10^{-3}
- On a Casio calculator the button you will need to type in numbers in scientific notation is:

- To type in 7 x 10^4, press "7" and the button above and then "4"
- To convert numbers into scientific notation on your calculator:
  - Type in the number and press = to enter it on the screen.
  - Press "Shift" + "Mode" and select "Sci" from the menu. Then press "0".

d) Foreign Exchange

Steps:

1. Write the conversion with the currency you want on the right.
2. Get a 1 on the left-hand side, by dividing both sides.
3. Multiply both sides to get the value you want.

Example: If €1 = $1.32, how many euro would you get for $200?

Step 1: Put euro on the right

$1.32 = €1

Step 2: Get a 1 on the left-hand side

$1 = € $\frac{1}{1.32}$ (dividing both sides by 1.32)

Step 3: Multiply both sides

$200 = $\frac{1}{1.32}$ x 200 = €151.52

e) HCF/LCM using Prime Factors:

Notes:

- When asked to find the HCF and LCM of 2 numbers using **prime factors**, use your calculator.
- Type in the number first and then press Shift + Button below.

- For **HCF:** pick the **lowest** power of each prime number.
- For **LCM:** pick the **highest** power of each prime number.

Example: Find the HCF and LCM of 60 and 80.

Using calculator: 60 = $5^1 \times 3^1 \times 2^2$ 80 = $2^4 \times 5^1$

HCF:

- Lowest Power of 2 is 2^2
- Lowest Power of 3 is 3^1.....but 3^1 doesn't go into 80 so exclude
- Lowest Power of 5 is 5^1 => HCF of 60 and 80 is $2^2 \times 5^1$ = 10

LCM:

- Highest Power of 2 is 2^4
- Highest Power of 3 is 3^1
- Highest Power of 5 is 5^1 => LCM of 60 and 80 is $2^4 x 3^1 x 5^1$ = 240

f) Speed, Distance and Time:

Notes:

- For all speed, distance and time calculations remember:

"Dads Silly Triangle"

If you want **Distance**, you cover the D in the triangle, so:

D = S x T (Units are usually m or km)

- If you want **Time**, you cover the T in the triangle, so:

T = D / S (Units are usually secs or hrs)

- If you want **Speed**, you cover the S in the triangle, so:

S = D / T (Units might be m/s or km/h)

- Average speed can be calculated using:

$$\text{Ave Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

- **Careful with units of time.** E.g. 1hr 45mins = 1.75hrs etc.

g) Ratio

Notes:

- Ratio shows how to break up a quantity proportionally.
- When given a ratio, add the values in the ratio together to get the total number of parts the quantity is being broken into.
- Write down the fraction each person gets.

Example: Divide €200 between Alan and Brian in the ratio 3:2.

3:2 means there are 3 + 2 = 5 parts

=> Alan gets $\frac{3}{5}$ and Brian gets $\frac{2}{5}$ => Alan gets $\frac{3}{5}$ of €200 = €120 and Brian gets $\frac{2}{5}$ of €200 = €80

2) Percentages/Profit/Loss/VAT:

a) Percentages:

To find the percentage of a number:

Example: Find 24% of 250.

Method 1: Calculate $\frac{24}{100} x \frac{250}{1} = 60$

Method 2: Multiply 250 by 0.24 = 60

To find the total when given percentage:

Example: 25% of the marks in an exam are going for the practical part. If there are 50 marks for the practical, how many marks is the whole exam worth?

Steps:

1) Let % = value you're given
   25% = 50
2) Find what 1% represents by dividing both sides
   1% = $\frac{50}{25} = 2$
3) Find 100% by multiplying by 100:
   100% = 2 x 100 = 200marks

Note:

In this particular example, we could also have just multiplied 50 by 4, as 25% represents $^1/_4$ of the total marks

b) % Profit / Loss / Discount:

$$\% \text{ Profit or Mark-Up} = \frac{Profit}{Cost\ Price} x\ 100\ \%$$

$$\% \text{ Loss} = \frac{Loss}{Cost\ Price} x\ 100\ \%$$

$$\% \text{ Discount} = \frac{Discount}{Cost\ Price} x\ 100\ \%$$

c) VAT:

VAT excluded:

Example: Bill comes to €120. Find final bill with 13.5% VAT.

VAT = 13.5% of 120
= 120 x 0.0135 = €16.20
=> Final Bill = €120 + €16.20 = €136.20

VAT included:

Example: Bill including VAT comes to €340.50. Find bill without VAT, if VAT is 13.5%.

Bill + VAT = €340.50
=> 113.5% = €340.50
=> 1% = €3
=> 100% = €300

3) Income Tax:

a) Income Tax Terminology:

- **Gross Income:** total pay someone gets before any taxes or deductions are taken
- **Net Income:** Take home pay
- **Standard** Rate of Tax (usually about 20%)
- **Gross Tax:** Total tax owing to the government before credits are deducted
- **Tax Credits:** Money deducted from the gross tax
- **Tax Payable:** Tax that you actually pay

b) Answering Questions:

- The questions are nearly always made up of 3 parts:

- **Part 1:** Calculation of Gross Tax
- **Part 2:** Calculation of Tax Payable using the equation
  Tax Paid = Gross Tax - Tax Credits
- **Part 3:** Working out Net Income by taking off all deductions including Tax Paid.

4) Compound Interest/Depreciation:

a) Terminology:

- **Principal:** Amount of money invested or borrowed
- **Interest:** Money added by the bank
- **Rate:** what percentage the interest is added at
- **Amount or Final Value:** The value of money at the end of the term it has been borrowed or invested for.

b) Answering Compound Interest Questions:

Method 1: Used if rates change from year to year or payments/withdrawals are being made between years

- Lay out Year 1, Year 2, Year 3 etc.
- Work out interest each year and add to Principal at start of the year

Method 2: Formula

$$F = P(1+i)^t$$

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where **F** is the Amount, **P** is the Principal, **i** is the Rate of Interest as a **decimal** (e.g. 3% = 0.03) and **t** is the time in years the money is invested/borrowed for.

5) Household Bills:

Notes:

- With utility bills (e.g.s. gas, electricity, water) there is usually a unit rate i.e. a charge per unit used
- To calculate the units used, subtract the previous units reading from the current units reading
- With many bills there is also a standing charge that has to be added on.
- VAT is also added to the bills.
- With Gas Bills, there is also a Carbon Tax that needs to be added on.

Example: Calculate the cost of electricity if the previous meter reading was 21310 and the current reading is 21836, with a standing charge of €21.60. The cost per unit is €0.15 and VAT of 13.5% is added on.

Units used = Current Reading - Previous Reading
= 21836 – 21310 = 526 units
Cost for electricity = 526 x €0.15 = €78.90
Standing Charge = €21.60
=> Total Before VAT = €78.90 + €21.60 = €100.50
VAT = 13.5% of €100.50 = €13.57
=> Final Bill = €100.50 + €13.57 = €114.07