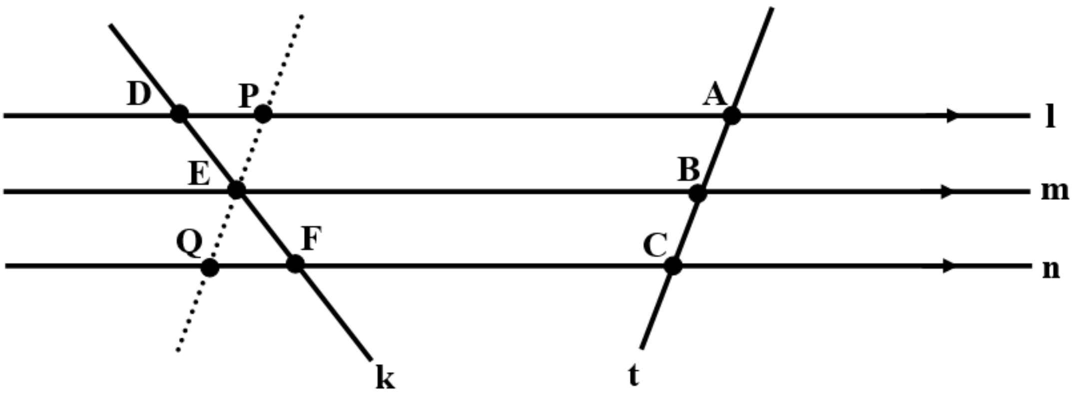
Theorem 11: If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.

Diagram:



Given: Three parallel lines l, m and n, intersecting the transversal t at the points A, B and C such that |AB| = |BC|. Another transversal k intersects the lines at D, E and F.

To Prove: |DE| = |EF|

Construction: Through E, construct a line parallel to t and intersecting l at the point P and n at the point Q.

Proof: PEBA and EQCB are parallelograms (from construction)

=> |PE| = |AB| and |EQ| = |BC| (opposite sides of a parallelogram)

But |AB| = |BC| (Given)

So => |PE| = |EQ|.

Consider now $\triangle$DEP and $\triangle$FEQ:

|PE| = |EQ| (from above)

|$\angle$ PED| = |$\angle$FEQ| (Vertically opposite angles)

|$\angle$DPE| = |$\angle$FQE| (Alternate angles)

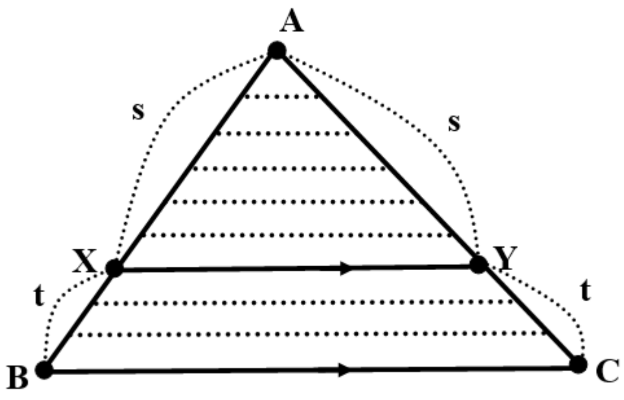
=> $\triangle$DEP and $\triangle$FEQ are congruent (ASA)

=> |DE| = |EF|.

Q.E.D.

Theorem 12: Let ABC be a triangle. If a line XY is parallel to BC and cuts [AB] in the ratio s : t, then it also cuts [AC] in the same ratio.

Diagram:



Given: The triangle ABC with XY parallel to BC.

To Prove: $\dfrac{|AX|}{|XB|} = \dfrac{|AY|}{|YC|}$

Construction: Divide [AX] into s equal parts and [XB] into t equal parts.

Draw a line parallel to BC through each point of division.

Proof: The parallel lines make intercepts of equal length along the line [AC] (From Theoerem 11)

=> [AY] is divided into s equal intercepts and [YC] is divided into t equal intercepts.

=> $\dfrac{|AY|}{|YC|} = \dfrac{s}{t}$

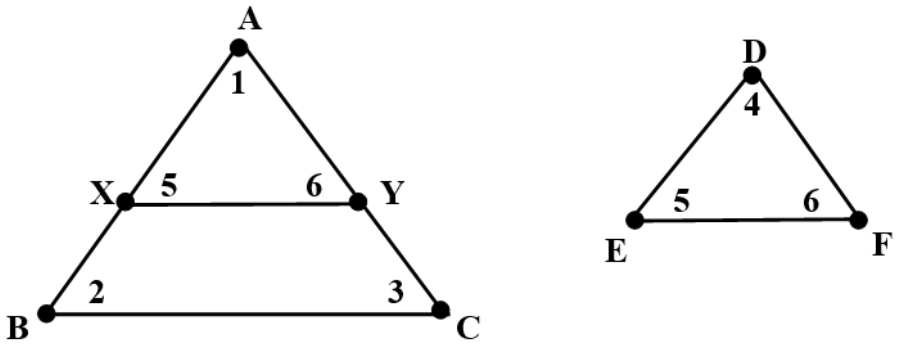
But $\dfrac{|AX|}{|XB|} = \dfrac{s}{t}$

=> $\dfrac{|AX|}{|XB|} = \dfrac{|AY|}{|YC|}$

Q.E.D.

Theorem 13: If two triangles ABC and DEF are similar, then their sides are proportional in order.

Diagram:



Given: Similar triangles ABC and DEF

To prove: $\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$

Construction: Mark the point X on [AB] such that |AX| = |DE|.

Mark the point Y on [AC] such that |AY| =|DF|.

Join XY.

Proof: $\triangle$ AXY and $\triangle$ DEF are congruent. (SAS)

=> $|\angle AXY| = |\angle DEF| = |\angle 5|$ (Corresponding angles)

=> $|\angle AXY| = |\angle ABC|$ (as triangles ABC and DEF are similar)

=> $XY \parallel BC$ (as angles 2 and 5 are corresponding)

=> $\frac{|AB|}{|AX|} = \frac{|AC|}{|AY|}$ (A line parallel to one side divides other side in the same ratio….Theorem 12)

=> $\frac{|AB|}{|DE|} = \frac{|AC|}{|DF|}$

Similarly, it can be proven that $\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|}$

=> $\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$ Q.E.D.