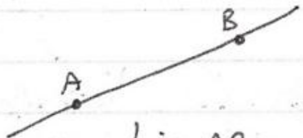
Leaving Cert Geometry: (Higher Level)

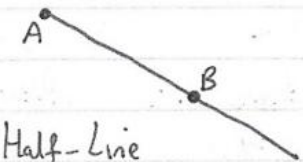
- ## Section 1A: Junior Cert Lines/Angles:

Lines/Angles:

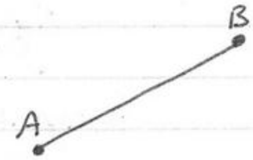
* Types of Lines:



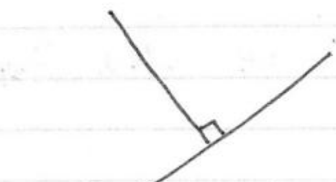
Line AB



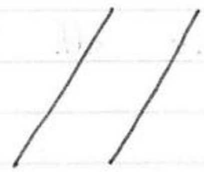
Half-Line or Ray [AB



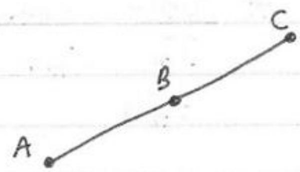
Line Segment [AB]



Perpendicular Lines

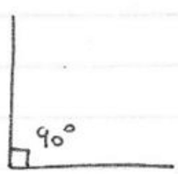


Parallel Lines

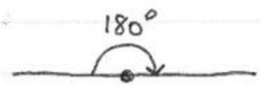


Collinear Points

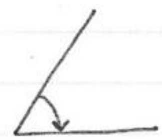
* Types of Angles:



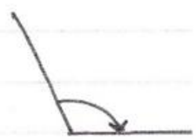
Right Angle



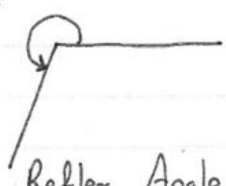
Straight Angle



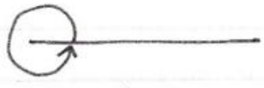
Acute Angle (between 0° and 90°)



Obtuse Angles (between 90° & 180°)

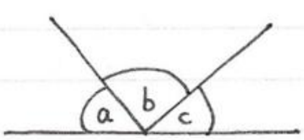


Reflex Angle (between 180° and 360°)

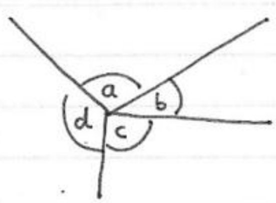


Full Angle (360°)

* Combining Angles:

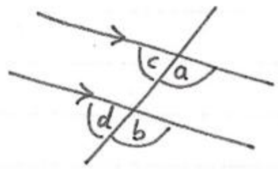


$a+b+c = 180°$

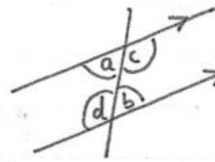


$a+b+c+d = 360°$

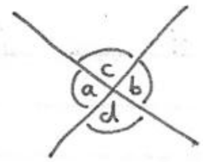
* Equal pairs of angles:



Corresponding Angles (F)
$(a = b)$ $(c = d)$



Alternate Angles (Z)
$(a = b)$ $(c = d)$

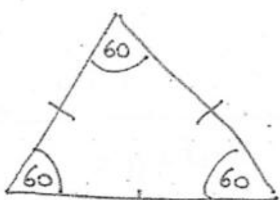


Vertically Opposite Angles
$(a = b)$ $(c = d)$

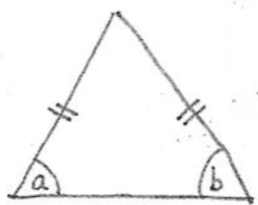
* Other angles:
Two angles that add to 180° are called Supplementary Angles.

- Section 1B: Junior Cert Triangles/Transversals:

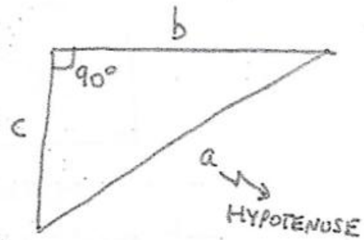
i) Types of Triangles:



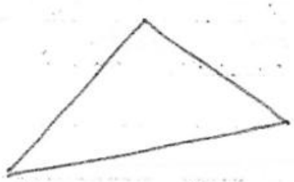
Equilateral Δ
(All sides/angles equal)



Isosceles Δ
($a = b$ @ base of 2 equal sides)



Right Angled Δ
$(a^2 = b^2 + c^2)$

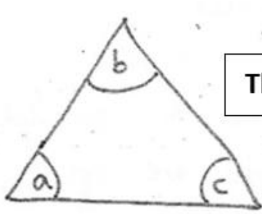


Scalene Δ
(No equal sides/angles)

* Area of a Δ = $\frac{1}{2}$ base × h
* Area of a Δ = $\frac{1}{2} ab \sin C$

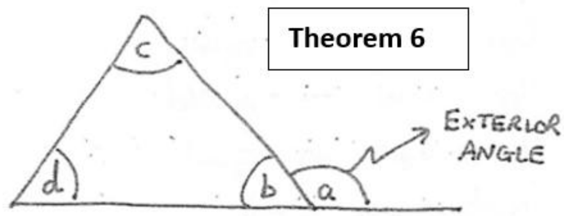
ii) Triangle Properties:

Theorem 4



$a + b + c = 180°$

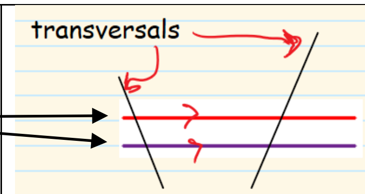
Theorem 6



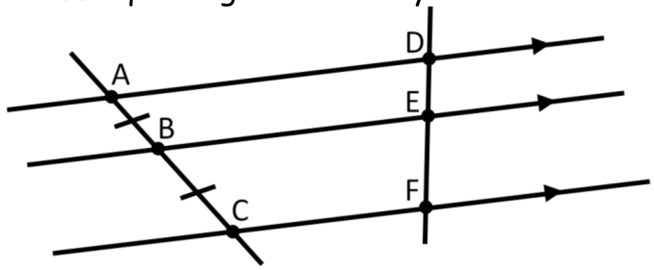
$a = c + d$

iii) Transversals:

- A straight line that intersects two or more lines.
- Note: The two lines don't have to be parallel.



- **Theorem:** If 3 parallel lines cut off equal segments on a transversal, then they cut off equal segments on any other transversal.

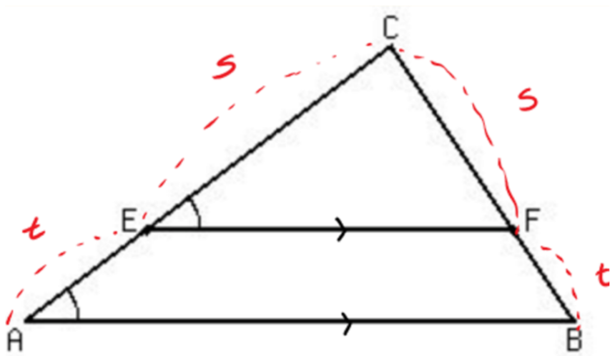


If $|AB| = |BC|$

=> $|DE| = |EF|$

iv) Line parallel to 1 side of a triangle:

- **Theorem:** A line parallel to 1 side of a triangle, cuts the other 2 sides in the same ratio.



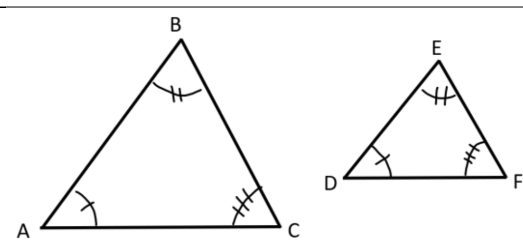
If EF is parallel to AB

=> $\frac{|AE|}{|CE|} = \frac{|BF|}{|CF|}$

or $\frac{|AE|}{|AC|} = \frac{|BF|}{|BC|}$

v) Similar (Equiangular) Triangles:

- Two triangles with identical angles.
- Symbol: ~
- The matching sides are called Corresponding sides E.g. |AB| and |DE| are corresponding

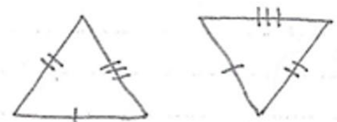


- **Theorem:** If 2 triangles are similar, then their corresponding sides are proportional, in order.
- In the triangles above:

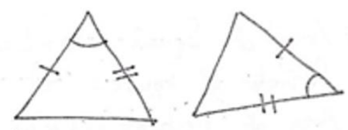
$$\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|} \text{ or } \frac{|DE|}{|AB|} = \frac{|EF|}{|BC|} = \frac{|DF|}{|AC|}$$

vi) Congruent Triangles:

- Two triangles that are identical (can sit exactly on top of each other.
- Symbol: ≡
- Matching sides are also called Corresponding sides
- 4 types:

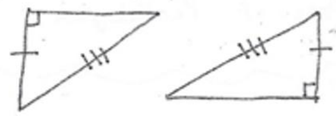


3 Sides Equal (SSS)



2 Sides equal + Angle In Between (SAS)

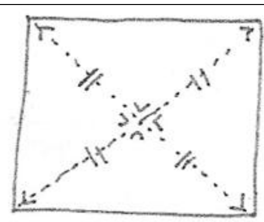
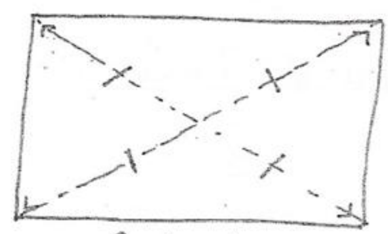
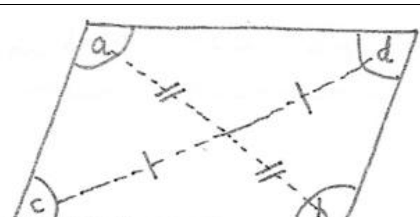
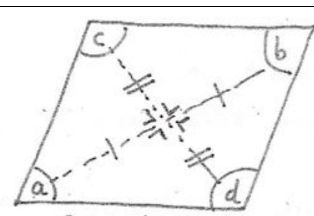
2 angles equal + Side in between (ASA)



Right Angle + Hypotenuse + Another side (RHS)

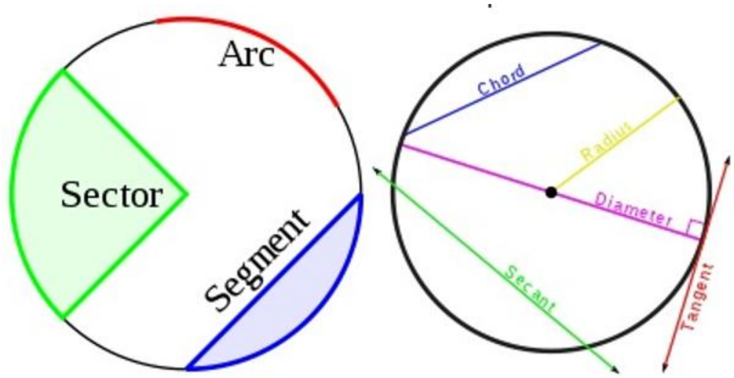
➢ Section 1C: Junior Cert Quadrilaterals:

Types of Quadrilaterals and their properties:

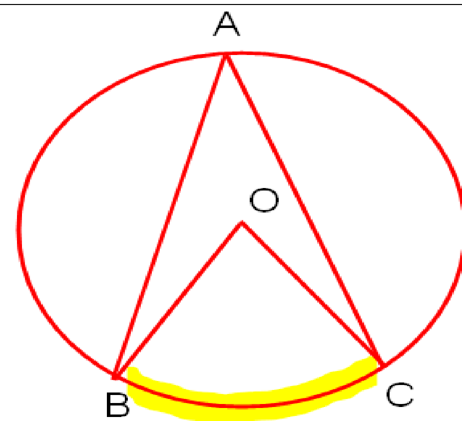
 Square – 4 equal sides – Opp sides are parallel – 4 equal angles (90°) – Diagonals bisect and make 90° angles	 Rectangle – Opp sides are equal – Opp sides are parallel – 4 equal angles (90°) – Diagonals bisect
 Parallelogram Theorem 9 – Opp sides are equal – Opp sides are parallel – Opp angles are equal (a=b and c=d) – Diagonals bisect	 Rhombus – 4 equal sides – Opp sides are parallel – Opp angles are equal (a=b and c=d) – Diagonals bisect and make 90° angles

Section 1D: Junior Cert Circles:

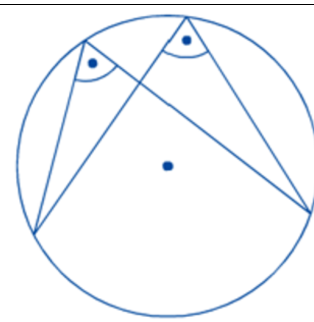
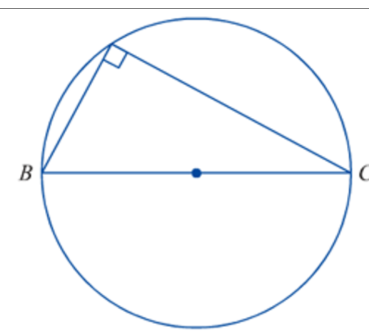
a) Terminology:



b) Angle at the centre of a circle:

○ **Theorem:** The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc. $$\|<BOC\| = 2(\|<BAC\|)$$	

c) Angle on the same arc/Angle in a semi-circle:

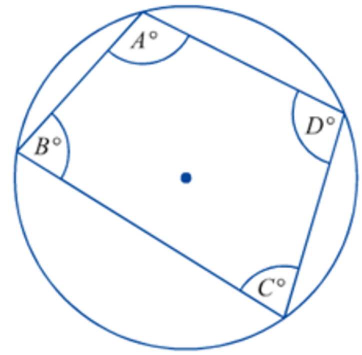
○ 3 results that follow from the previous theorem:	
○ **Corollary 1:** All angles at points of a circle standing on the same arc are equal. ○ Note: Watch for the "butterfly wings" shape.	
○ **Corollary 2:** Each angle in a semi-circle is a right angle. ○ It also applies that……if the angle standing on the chord [BC] at some point on the circle is a right angle, then [BC] must be a diameter…. **Corollary 3:**	

d) Cyclic Quadrilaterals:

- A 4 sided shape with its 4 vertices on a circle.
- A final result that follows from the previous theorem is:
- **Corollary 4:** In a cyclic quadrilateral, the opposite angles add to 180°.

A + C = 180°

B + D = 180°

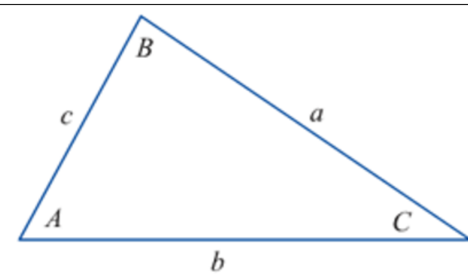


Section 2: Additional Triangle Properties:

a) Biggest/smallest angles:

- **Theorem:** The largest angle is opposite the largest side in a triangle. The smallest angle is opposite the smallest side.

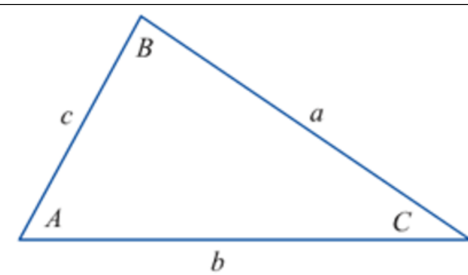
If a is the largest side, then A is the largest angle, and vice versa.



b) Triangle inequality:

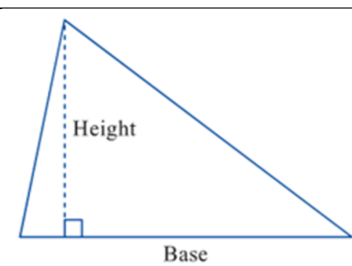
- **Theorem:** Two sides of a triangle are together greater than the third.

$$a + b > c$$
$$b + c > a$$
$$a + c > b$$



c) Area of a Triangle:

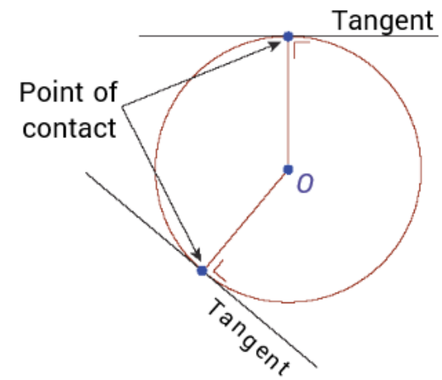
- **Theorem:** For a triangle, base x height does not depend on the choice of base.



- Section 3: Additional Circle Geometry:

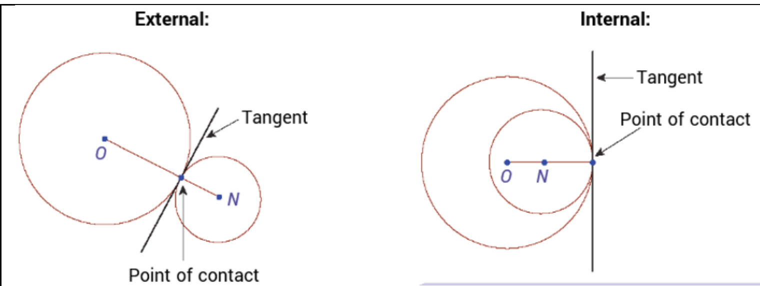
a) Tangent to a circle:

- A tangent is a line that touches a circle at one point only.
- **Theorem:** A tangent to a circle is at a right angle to the radius at the point of contact.
- The **converse** also applies….if a line is perpendicular to a radius at a point P on the circle, then the line must be a tangent.



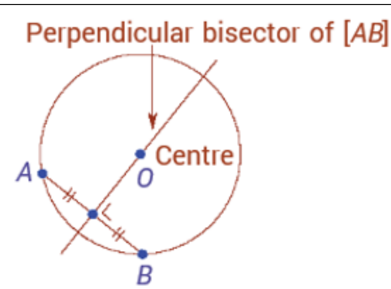
b) Touching Circles:

- A result that follows from the previous theorem is:
- **Corollary:** If two circles intersect at 1 point only, then the 2 centres and the point of contact are collinear.
- Circles can touch externally or internally.

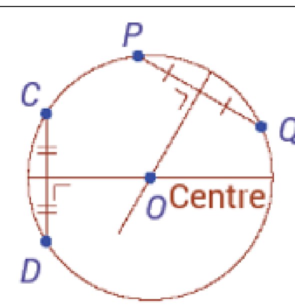


c) Chord Bisectors:

- **Theorem:** The perpendicular from the centre of a circle to a chord bisects the chord.

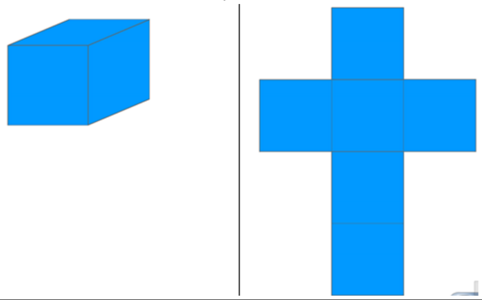


- **Theorem:** The perpendicular bisector of a chord passes through the centre of a circle.

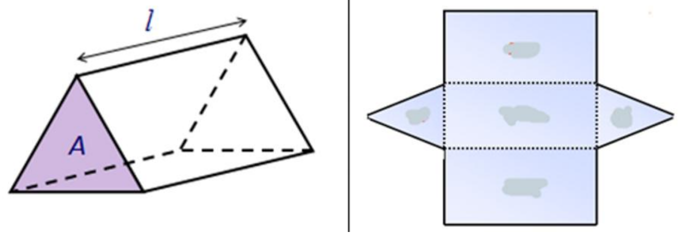


➢ Section 4: Nets of 3D Shapes:

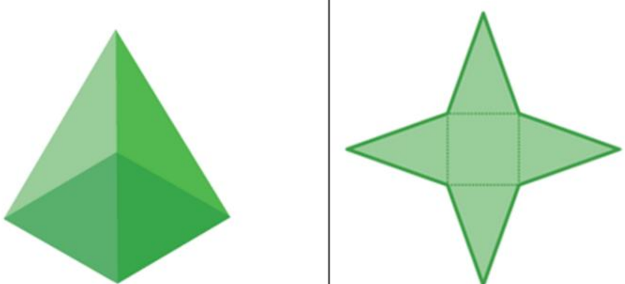
Cube/Cuboid



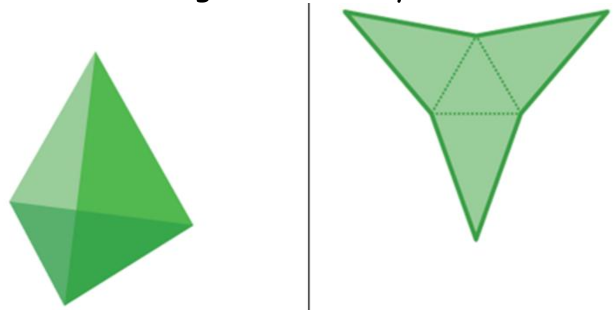
Triangular Prism



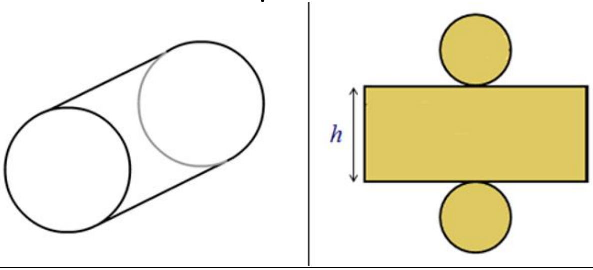
Square Based Pyramid



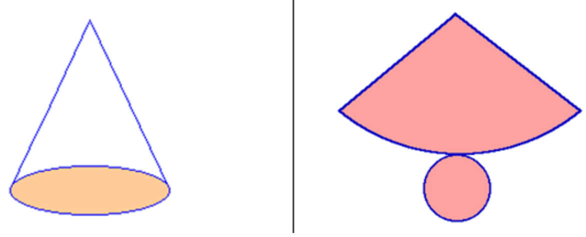
Triangular Based Pyramid



Cylinder



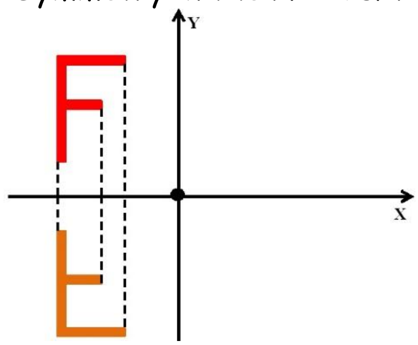
Cone



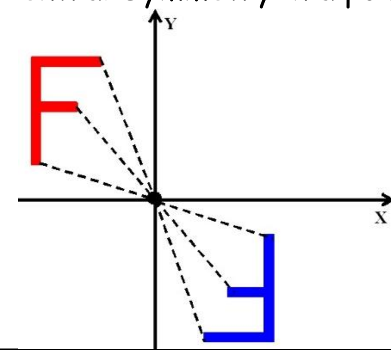
- Section 5: Recap on Transformation Geometry:

a) Types of Transformations:

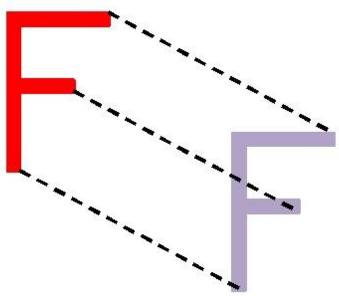
Axial Symmetry in the X-Axis/Y-Axis



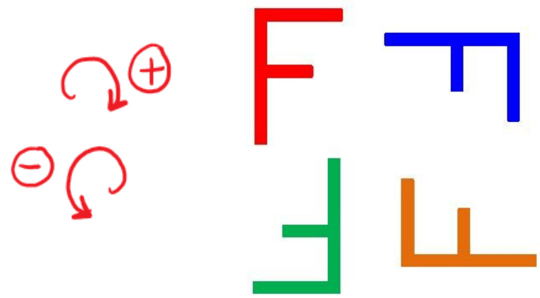
Central Symmetry in a point



Translations

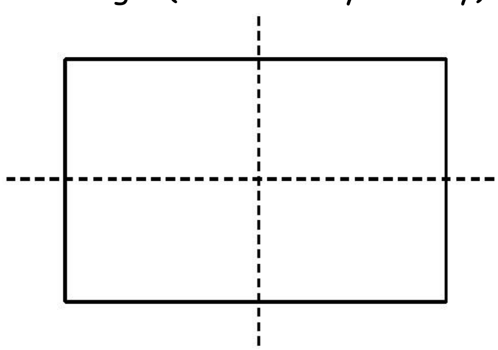


Rotations

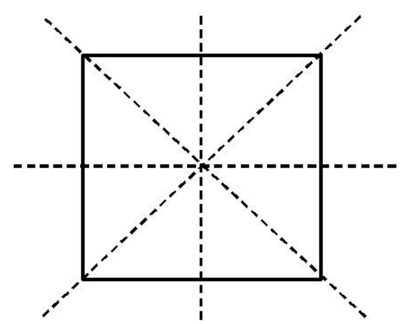


b) Axes of symmetry of a shape:

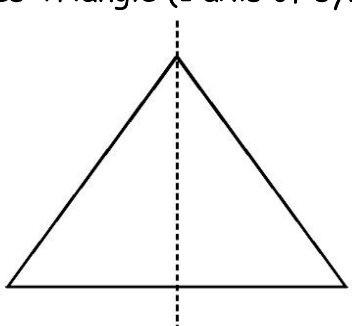
Rectangle (2 axes of symmetry)



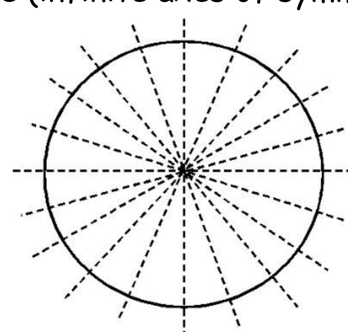
Square (4 axes of symmetry)



Isosceles Triangle (1 axis of symmetry)



Circle (infinite axes of symmetry)



- ## Section 6: Theorem Proofs:

a) Terminology of Theorems:

- **Axiom:** An axiom is a statement that we accept without any proof.
  - Example: There is exactly one line through two points.
- **Theorem:** A theorem is a rule that you can prove using a certain number of logical steps, using previously established theorems or axioms.
  - Example: The sum of the angles in a triangle is equal to 180°.
- **Proof:** A proof is s set of logical steps that we use to prove a theorem.
- **Corollary:** A corollary is a statement that follows on from a previous theorem.
  - Example: In Theorem 19 we prove that the angle at the centre of a circle is twice the measure of the angle at the circumference standing on the same arc. A corollary of this is that the angle on a semi-circle is 90°.
- **Converse:** The converse of a statement is formed by reversing the order in which the statement is made.
  - For example: Statement: In a right-angled triangle $H^2 = O^2 + A^2$. Converse: If $H^2 = O^2 + A^2$ in a triangle, then the triangle must be right angled.
- **Implies:** Implies is a term we use in proofs, when we write a statement of fact that follows from a previous statement. The symbol we use for implies is =>.
  - Example: John spends a lot of time watching TV
    => John must like watching TV!

b) Proof by contradiction:

* Assume opposite is true and prove that the opposite is impossible.

- Example 1:

Prove that an equilateral Δ is also an acute-angled Δ (i.e. has no angle bigger than 90°)

Proof:

Assume opposite true i.e. equilateral Δ is NOT acute-angled
$\Rightarrow$ it has one angle bigger than 90°

As Δ is equilateral, all angles are equal
$\Rightarrow$ it has 3 angles bigger than 90°

But...

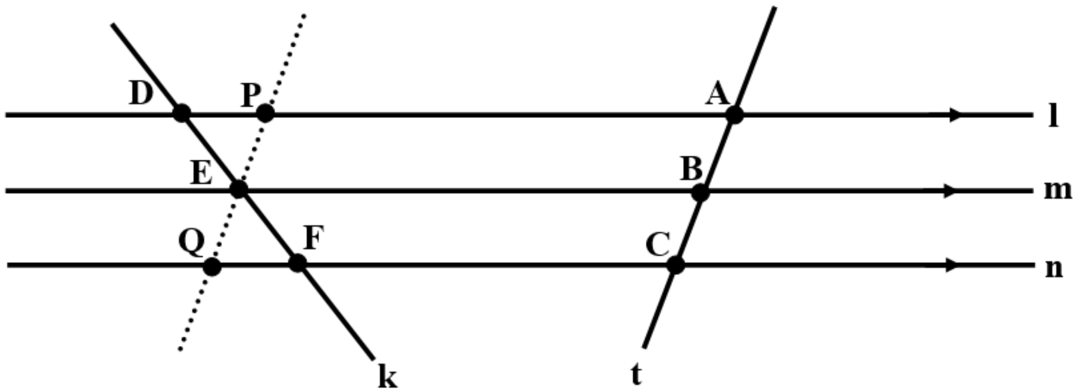
Sum of 3 angles = 180° so this is impossible!

c) <u>The 3 theorems on the course that need to be proved:</u>

i)

Theorem 11: If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.

Diagram:



Given: Three parallel lines l, m and n, intersecting the transversal t at the points A, B and C such that |AB| = |BC|. Another transversal k intersects the lines at D, E and F.

To Prove: |DE| = |EF|

Construction: Through E, construct a line parallel to t and intersecting l at the point P and n at the point Q.

Proof: PEBA and EQCB are parallelograms (from construction)

=> |PE| = |AB| and |EQ| = |BC| (opposite sides of a parallelogram)

But |AB| = |BC| (Given)

So => |PE| = |EQ|.

Consider now $\triangle$DEP and $\triangle$FEQ:

|PE| = |EQ| (from above)

|$\angle$ PED| = |$\angle$ FEQ| (Vertically opposite angles)

|$\angle$ DPE| = |$\angle$ FQE| (Alternate angles)

=> $\triangle$DEP and $\triangle$FEQ are congruent (ASA)

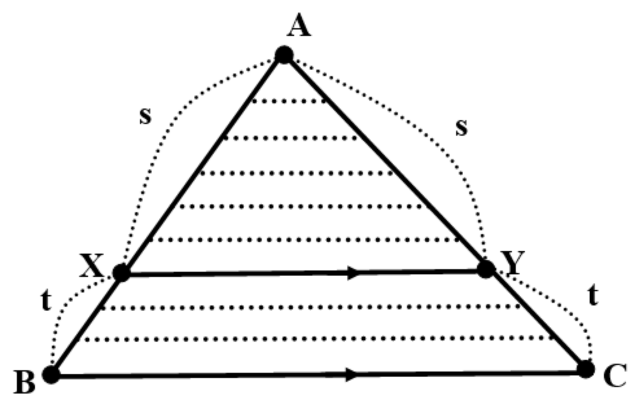
=> |DE| = |EF|.

Q.E.D.

ii)

Theorem 12: Let ABC be a triangle. If a line XY is parallel to BC and cuts [AB] in the ratio s : t, then it also cuts [AC] in the same ratio.

Diagram:



Given: The triangle ABC with XY parallel to BC.

To Prove: $\dfrac{|AX|}{|XB|} = \dfrac{|AY|}{|YC|}$

Construction: Divide [AX] into s equal parts and [XB] into t equal parts.

Draw a line parallel to BC through each point of division.

Proof: The parallel lines make intercepts of equal length along the line [AC] (From Theoerem 11)

=> [AY] is divided into s equal intercepts and [YC] is divided into t equal intercepts.

=> $\dfrac{|AY|}{|YC|} = \dfrac{s}{t}$

But $\dfrac{|AX|}{|XB|} = \dfrac{s}{t}$

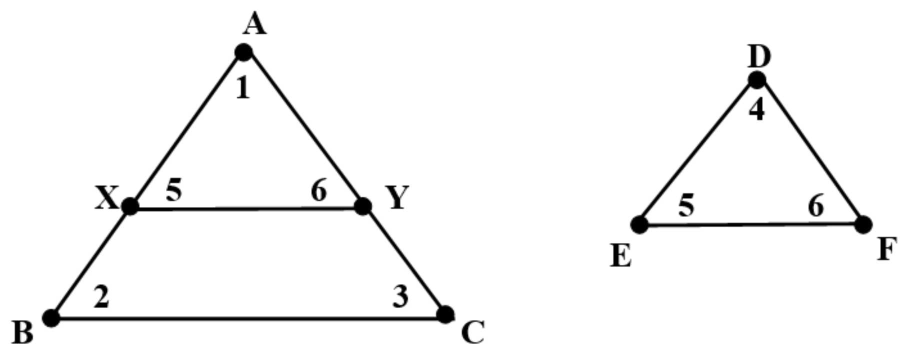
=> $\dfrac{|AX|}{|XB|} = \dfrac{|AY|}{|YC|}$

Q.E.D.

iii)

Theorem 13: If two triangles ABC and DEF are similar, then their sides are proportional in order.

Diagram:



Given: Similar triangles ABC and DEF

To prove: $\dfrac{|AB|}{|DE|} = \dfrac{|BC|}{|EF|} = \dfrac{|AC|}{|DF|}$

Construction: Mark the point X on [AB] such that |AX| = |DE|.

Mark the point Y on [AC] such that |AY| =|DF|.

Join XY.

Proof: $\triangle$ AXY and $\triangle$ DEF are congruent. (SAS)

=> $|\angle AXY| = |\angle DEF| = |\angle 5|$ (Corresponding angles)

=> $|\angle AXY| = |\angle ABC|$ (as triangles ABC and DEF are similar)

=> $XY \parallel BC$ (as angles 2 and 5 are corresponding)

=> $\dfrac{|AB|}{|AX|} = \dfrac{|AC|}{|AY|}$ (A line parallel to one side divides other side in the same ratio….Theorem 12)

=> $\dfrac{|AB|}{|DE|} = \dfrac{|AC|}{|DF|}$

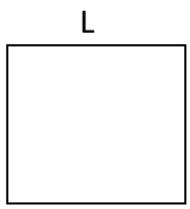
Similarly, it can be proven that $\dfrac{|AB|}{|DE|} = \dfrac{|BC|}{|EF|}$

=> $\dfrac{|AB|}{|DE|} = \dfrac{|BC|}{|EF|} = \dfrac{|AC|}{|DF|}$ Q.E.D.

- Section 7: Area/Volume Formulae:

N.B. The formulae circled below are **NOT** in the Tables book.

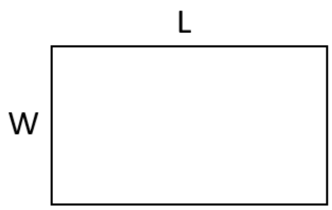
Square:



Area = L^2

Per = 4L

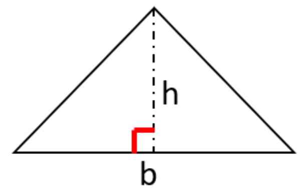
Rectangle:



Area = L x W

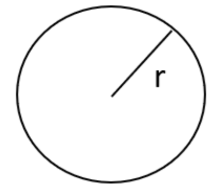
Per = 2L + 2W

Triangle:



Area = ½ b x h

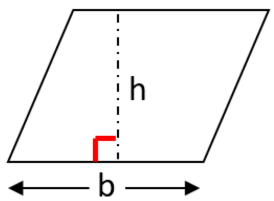
Circle:



Area = πr^2

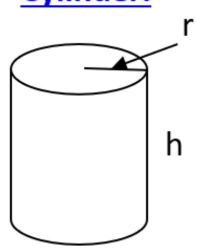
Circum = $2\pi r$

Parallelogram:



Area = b x h

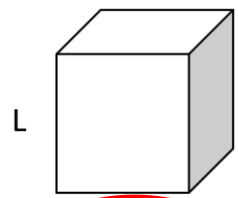
Cylinder:



Vol = $\pi r^2 h$

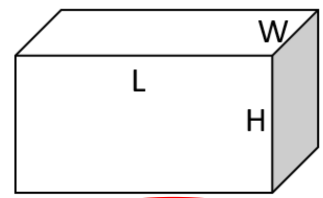
CSA = $2\pi rh$

Cube / Cuboid:



Vol = L^3

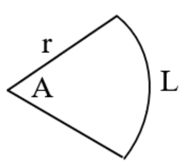
TSA = $6L^2$



Vol = L x W x H

TSA = 2LW + 2WH + 2HL

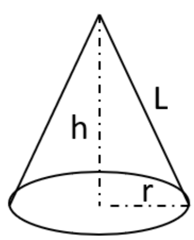
Sector:



Area = $\frac{A}{360^\circ}$ X πr^2

L = $\frac{A}{360^\circ}$ X $2\pi r$

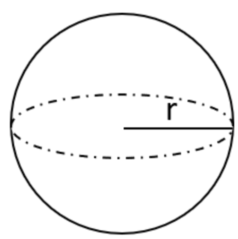
Cone:



Vol = $\frac{1}{3}\pi r^2 h$

CSA = πrL

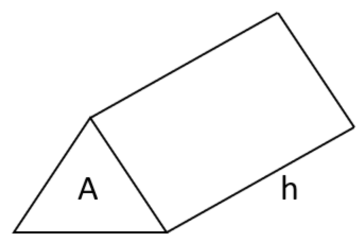
Sphere:



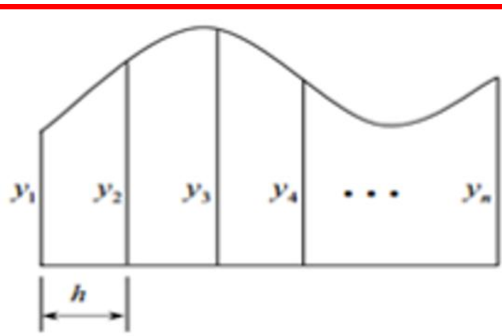
Vol = $4/3\pi r^3$

CSA = $4\pi r^2$

Prism:



Vol = Area A x h



Trapezoidal rule:

Area $\approx \frac{h}{2}\left[y_1 + y_n + 2\left(y_2 + y_3 + y_4 + \cdots + y_{n-1}\right)\right]$

or $\approx \frac{h}{2}\left[\text{first} + \text{last} + \text{twice the rest}\right]$