Worked Solutions

Q1. i)

Dist in 8^{th} sec = 5·625

$\Rightarrow S_8 - S_7 = 5·625$

$S_8 = ut + \frac{1}{2}at^2$

$= (0)(8) + \frac{1}{2}a(8)^2$

$= 32a$

$S_7 = ut + \frac{1}{2}at^2$

$= (0)(7) + \frac{1}{2}(a)(7)^2$

$= \frac{49a}{2}$

$S_8 - S_7 = 32a - \frac{49a}{2}$

$= \frac{64a}{2} - \frac{49a}{2}$

$= \frac{15a}{2}$

$\Rightarrow \frac{15a}{2} = 5·625$

$15a = 11·25$

$a = \frac{11·25}{15} = \boxed{0·75\ m/s^2}$

ii) $u = 0 \quad t = 20 \quad a = 0·75 \quad s = ?$

$s = ut + \frac{1}{2}at^2$

$s = (0)(20) + \frac{1}{2}(0·75)(20)^2$

$s = \boxed{150m}$

Q2

Sec 1:

$u = 0 \quad v = 20 \quad a = 2 \quad s = ?$

$v^2 = u^2 + 2as$

$(20)^2 = (0)^2 + 2(2)s$

$4s = 400$

$s = 100\,m$

$v = u + at$

$20 = 0 + 2t$

$\Rightarrow t_1 = 10s$

Sec 3:

$u = 20 \quad v = 0 \quad a = -4 \quad s = ?$

$v^2 = u^2 + 2as$

$(0)^2 = (20)^2 + 2(-4)s$

$8s = 400$

$s = 50m$

$v = u + at$

$0 = 20 - 4t$

$t_3 = 5s$

$\Rightarrow$ Time @ constant speed $= 60s - 10s - 5s$

$= 45s$

$\Rightarrow$ Dist @ constant speed $=$ Speed $\times$ Time

$= 20 \times 45$

$= 900\,m$

$\Rightarrow$ Tot Distance $= 100 + 900 + 50$

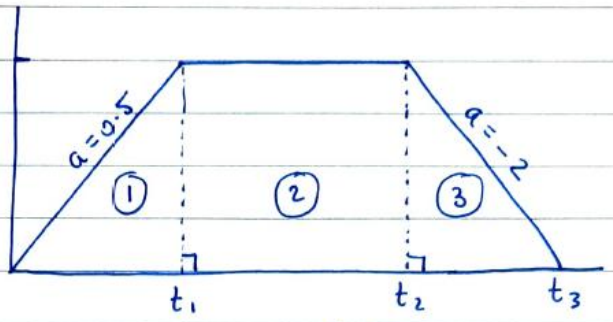
$= 1050\,m$

$\Rightarrow$ Ave Velocity $= \frac{\text{Tot Dist}}{\text{Tot Time}}$

$= \frac{1050}{60}$

$= \boxed{17.5\,m/s}$

Q3 i)



ii) Sec 1:

$u = 0 \quad a = 0.5 \quad s = 256 \quad v = ?$

$v^2 = u^2 + 2as$

$v^2 = (0)^2 + 2(0.5)(256)$

$v^2 = 256$

$v = \boxed{16 \text{ m/s}}$

$v = u + at$

$16 = 0 + 0.5(t_1)$

$\Rightarrow t_1 = 32\text{s}$

Sec 2

Area under graph = 800m

$16T = 800$

$T = 50\text{s}$

Sec 3

$u = 16 \quad v = 0 \quad a = -2 \quad t = ?$

$v = u + at$

$0 = 16 - 2t_3$

$t_3 = 8\text{s}$

iii) Tot Time $= 32 + 50 + 8$

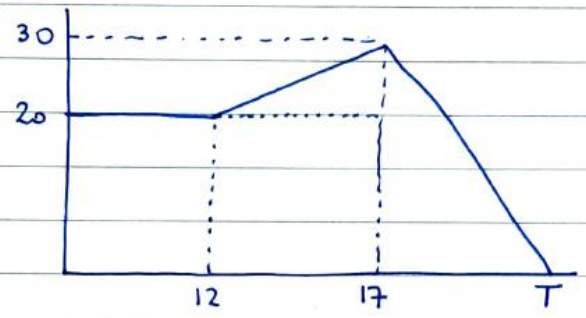
$= \boxed{90\text{s}}$

iv) Tot Dist $= \frac{1}{2}(32)(16) + 800 + \frac{1}{2}(8)(16)$

$= 256 + 800 + 64$

$= \boxed{1120\text{m}}$

Q4.



i) $u = 20 \quad v = 30 \quad a = ? \quad t = 5$

$v = u + at$

$30 = 20 + 5a$

$5a = 10$

$a = \boxed{2\ m/s^2}$

ii) $u = 30 \quad v = 0 \quad s = 75 \quad a = ?$

$v^2 = u^2 + 2as$

$(0)^2 = (30)^2 + 2(a)(75)$

$150a = -900$

$a = -6\ m/s^2$

$\Rightarrow$ dec = $\boxed{6\ m/s^2}$

$u = 30 \quad v = 0 \quad a = -6 \quad t = ?$

$v = u + at$

$0 = 30 - 6t$

$\Rightarrow t = 5s$

iii) Area under Graph = $(12)(20) + (5)(20) + \frac{1}{2}(5)(10) + \frac{1}{2}(5)(30)$

$= 240 + 100 + 25 + 75$

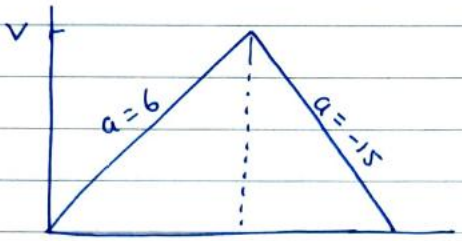
$= \boxed{440m}$

iv) Ave Speed = $\frac{\text{Tot Dist}}{\text{Tot Time}}$

$= \frac{440}{22}$

$= \boxed{20\ m/s}$

Q5



Method 1:

Sec 1: $u=0 \quad a=6 \quad v=v \quad t=t_1$

$v = u + at$

$v = 6t_1$

$\Rightarrow t_1 = v/6$

Sec 2: $u=v \quad v=0 \quad a=-15$

$v = u + at$

$0 = v - 15t_2$

$t_2 = v/15$

$\Rightarrow$ Tot Time $= \frac{v}{6} + \frac{v}{15} = \frac{7v}{30}$

$\Rightarrow$ Area Under $= \frac{1}{2}\left(\frac{7v}{30}\right)(v) = 1000$

$\Rightarrow 7v^2 = 60000$

$\Rightarrow v^2 = \frac{60000}{7}$

$\Rightarrow v = \sqrt{\frac{60000}{7}} = 92.582$

$\Rightarrow$ Tot Time $= \frac{7(92.582)}{30}$

$= \boxed{21.6\,s}$

Method 2:

$a : d = 6 : 15$

$\Rightarrow t_1 : t_2 = 15 : 6 = 5 : 2$

$\Rightarrow t_1 = \frac{5}{7}T \quad t_2 = \frac{2}{7}T$

Sec 1:

$u=0 \quad v=v \quad t=\frac{5}{7}T \quad a=6$

$v = 0 + 6\left(\frac{5}{7}T\right)$

$v = \frac{30T}{7}$

Area $= \frac{1}{2}\left(\frac{30T}{7}\right)(T) = 1000$

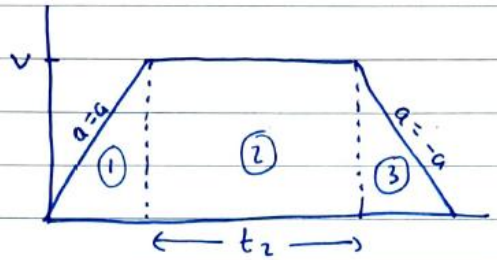
$\Rightarrow 30T^2 = 14000$

$\Rightarrow T^2 = \frac{14000}{30}$

$\Rightarrow T = \sqrt{\frac{14000}{30}}$

$= \boxed{21.6s}$

Q6.



i)

Sec 1	Sec 3
$u=0$ $v=v$ $a=a$ $t=t_1$	$u=v$ $v=0$ $a=-a$ $t=t_3$
$v = u+at$	$v = u+at$
$v = at_1$	$0 = v-at_3$
$\Rightarrow t_1 = v/a$	$\Rightarrow t_3 = v/a$

$$\Rightarrow \text{Total Time} = t = \frac{v}{a} + \frac{v}{a} + t_2$$

$$\Rightarrow at = 2v + at_2$$

$$2v = at - at_2$$

$$v = \boxed{\frac{a}{2}(t - t_2)} \quad \text{Q.E.D.}$$

ii) Area = s

$$\Rightarrow \frac{1}{2}\left(\frac{v}{a}\right)(v) + t_2(v) + \frac{1}{2}\left(\frac{v}{a}\right)(v) = s$$

$$\frac{v^2}{2a} + vt_2 + \frac{v^2}{2a} = s$$

$$2v^2 + 2avt_2 = 2as$$

$$v^2 + avt_2 = as$$

$$\left(\frac{a}{2}(t-t_2)\right)^2 + at_2\left(\frac{a}{2}(t-t_2)\right) = as$$

$$\frac{a^2}{4}(t^2 + t_2^2 - 2tt_2) + \frac{a^2t_2}{2}(t-t_2) = as$$

$$at^2 + at_2^2 - 2tt_2a + 2at_2(t-t_2) = 4s \quad (\div a \text{ and } \times 4)$$

$$at^2 + at_2^2 - 2tt_2a + 2at_2 - 2at_2^2 = 4s$$

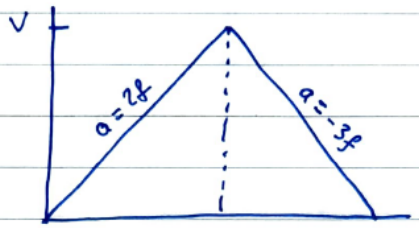
$$at^2 - at_2^2 = 4s$$

$$at_2^2 = at^2 - 4s$$

$$t_2^2 = \frac{at^2 - 4s}{a}$$

$$\Rightarrow \boxed{t_2 = \sqrt{\frac{at^2 - 4s}{a}}}$$

Q7



Sec 1

$u=0 \quad v=v \quad a=2f \quad t=t_1$

$v = u + at$

$v = 2f t_1$

$\Rightarrow t_1 = \frac{v}{2f}$

Sec 2

$u=v \quad v=0 \quad a=-3f \quad t=t_2$

$v = u + at$

$0 = v - 3f t_2$

$\Rightarrow t_2 = \frac{v}{3f}$

$\Rightarrow$ Total Time $= \frac{v}{2f} + \frac{v}{3f}$

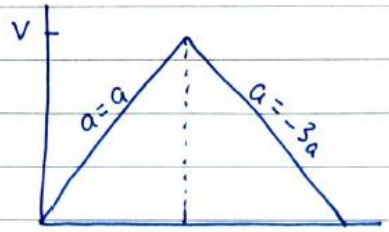
$t = \frac{5v}{6f} \Rightarrow v = \frac{6ft}{5}$

$\Rightarrow$ Area $= \frac{1}{2}(t)\left(\frac{6ft}{5}\right) = h$

$6ft^2 = 10h$

$\Rightarrow h = \frac{6ft^2}{10} = \boxed{\frac{3ft^2}{5}}$

Q8.



$a : d = 1 : 3$

$\Rightarrow t_1 : t_2 = 3 : 1$

$\Rightarrow t_1 = \frac{3}{4}T \quad t_2 = \frac{1}{4}T$

Sec 1

$u = 0 \quad v = v \quad a = a \quad t = \frac{3}{4}T$

$v = u + at$

$v = a\left(\frac{3}{4}T\right)$

$v = \frac{3aT}{4}$

Area Under Graph $= \frac{1}{2}(T)\left(\frac{3aT}{4}\right) = s$

$\Rightarrow \frac{3aT^2}{8} = s$

$\Rightarrow 3aT^2 = 8s \quad : I$

Ave Speed $= \frac{\text{Tot Dist}}{\text{Tot Time}} = \frac{s}{T}$

$\Rightarrow \frac{\frac{3aT^2}{8}}{T} = \sqrt{\frac{s}{2}}$

$\Rightarrow \left(\frac{3aT}{8}\right)^2 = \left(\sqrt{\frac{s}{2}}\right)^2$

$\frac{9a^2T^2}{64} = \frac{s}{2}$

$18a^2T^2 = 64s$

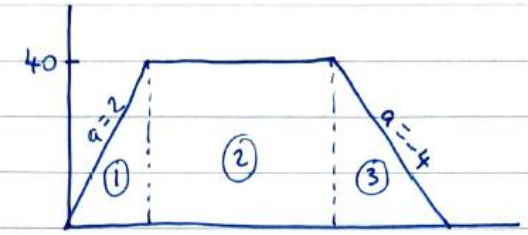
$6a(3aT^2) = 64s \quad$ (Using I)

$6a(8s) = 64s$

$48as = 64s$

$\Rightarrow a = \frac{64}{48} = \boxed{\frac{4}{3}\ m/s^2}$

Q9. i)



Sec 1

$u=0 \quad v=40 \quad a=2 \quad t=t_1$

$v = u+at$

$40 = 2t_1$

$t_1 = 20s$

Sec 3

$u=40 \quad v=0 \quad a=-4 \quad t=t_2$

$v = u+at$

$0 = 40-4t_2$

$t_2 = 10s$

Area Under Graph = 1000m

$\Rightarrow \frac{1}{2}(20)(40) + (t_2)(40) + \frac{1}{2}(10)(40) = 1000$

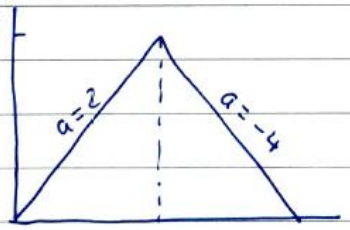
$400 + 40t_2 + 200 = 1000$

$40t_2 = 400$

$t_2 = 10s$

$\Rightarrow$ Tot Time = $20+10+10 =$ **40s**

ii)



$a:d = 1:2$

$\Rightarrow t_1 : t_2 = 2:1$

$\Rightarrow t_1 = \frac{2}{3}T \quad t_2 = \frac{1}{3}T$

Sec 1

$u=0 \quad v=v \quad a=2 \quad t=\frac{2}{3}T$

$v = u+at$

$v = 2\left(\frac{2T}{3}\right)$

$v = \frac{4T}{3}$

Area Under = 384

$\frac{1}{2}(T)\left(\frac{4T}{3}\right) = 384$

$\frac{4T^2}{6} = 384$

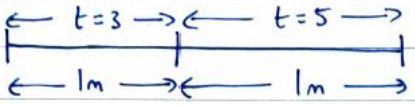
$4T^2 = 2304$

$T^2 = 576$

$T = \sqrt{576}$

$= 24s$

Q10.



Sec 1:

$u = u \quad a = a \quad t = 3 \quad s = 1$

$s = ut + \frac{1}{2}at^2$

$1 = 3u + \frac{1}{2}a(3)^2$

$\Rightarrow 6u + 9a = 2 \quad : I$

Sec 1 & 2:

$u = u \quad a = a \quad t = 8 \quad s = 2$

$s = ut + \frac{1}{2}at^2$

$2 = 8u + \frac{1}{2}a(8)^2$

$\Rightarrow 8u + 32a = 2 \quad II$

Solving I & II:

$I \times 4: \quad 24u + 36a = 8$

$II \times -3: \quad -24u - 96a = -6$

$-60a = 2$

$a = \boxed{-\frac{1}{30}\ m/s^2}$

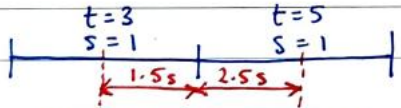
$I: \quad 6u + 9\left(-\frac{1}{30}\right) = 2$

$6u = 2 + \frac{3}{10}$

$6u = \frac{23}{10}$

$u = \boxed{\frac{23}{60}\ m/s}$

OR



Ave Speed $= \frac{1}{3}$ Ave Speed $= \frac{1}{5}$

$u = \frac{1}{3} \quad v = \frac{1}{5} \quad t = 1.5 + 2.5 = 4s \quad a = ?$

$v = u + at$

$\frac{1}{5} = \frac{1}{3} + 4a$

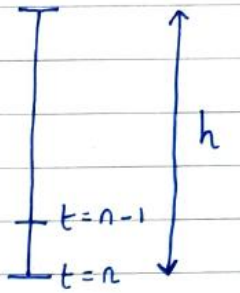
$4a = \frac{1}{5} - \frac{1}{3}$

$4a = -\frac{2}{15}$

$a = \boxed{-\frac{1}{30}\ m/s^2}$

Remake equation I from method 1 and solve for $u = \boxed{\frac{23}{60}\ m/s}$

Q11.



In n secs:

$u=0 \quad a=g \quad t=n \quad s=h$

$s = ut + \frac{1}{2}at^2$

$h = 0 + \frac{1}{2}(g)(n)^2$

$h = \frac{1}{2}gn^2 \quad : \text{I}$

In $n-1$ secs:

$u=0 \quad a=g \quad t=n-1 \quad s=\frac{3}{4}h$

$s = ut + \frac{1}{2}at^2$

$\frac{3}{4}h = 0 + \frac{1}{2}(g)(n-1)^2$

$\frac{3}{4}h = \frac{g}{2}(n^2-2n+1)$

$3h = 2g(n^2-2n+1) \quad (\times 4)$

$3h = 2gn^2 - 4gn + 2g \quad : \text{II}$

Put I into II:

$3(\frac{1}{2}gn^2) = 2gn^2 - 4gn + 2g$

$3gn^2 = 4gn^2 - 8gn + 4g$

$gn^2 - 8gn + 4g = 0$

$n^2 - 8n + 4 = 0$

$$n = \frac{8 \pm \sqrt{(-8)^2-4(1)(4)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{48}}{2}$$

$$= 4 \pm 2\sqrt{3}$$

$\Rightarrow$ Using I: $h = \frac{1}{2}g(4-2\sqrt{3})^2$ or $h = \frac{1}{2}g(4+2\sqrt{3})^2$

$= 1.4\text{m}$ $= 27.3\text{m}$

Height can't be 1.4m as stone would fall more than this height in 1s

$\Rightarrow$ Ans = $\boxed{27.3\text{m}}$

Q12.

i)

Let T = time to overtake

Car A

$u = 20 \quad a = 0 \quad t = T \quad s = S_1$

$S = ut + \frac{1}{2}at^2$

$S_1 = 20(T) + \frac{1}{2}(0)(T)^2$

$S_1 = 20T$

Car B

$u = 8 \quad a = 1.5 \quad t = T \quad s = S_2$

$S = ut + \frac{1}{2}at^2$

$S_2 = 8(T) + \frac{1}{2}(1.5)(T)^2$

$S_2 = 8T + \frac{3}{4}T^2$

$S_1 = S_2$

$\Rightarrow 20T = 8T + \frac{3}{4}T^2$

$12T = \frac{3}{4}T^2$

$48T = 3T^2$

$3T^2 - 48T = 0$

$3T(T - 16) = 0$

$3T = 0 \quad \text{or} \quad T - 16 = 0$

$T = 0 \quad \text{or} \quad \boxed{T = 16\,s}$

$S_1 = 20(16)$

$= \boxed{320\,m}$

ii) Greatest Distance $\Rightarrow V_1 = V_2$

$V_1 = 20 + 0(T) \qquad V_2 = 8 + 1.5T$

$V_1 = 20$

$\Rightarrow 20 = 8 + 1.5T$

$1.5T = 12$

$T = \frac{12}{1.5} = 8\,s$

$S_1 = 20(8) = 160\,m \qquad S_2 = 8(8) + \frac{3}{4}(8)^2$

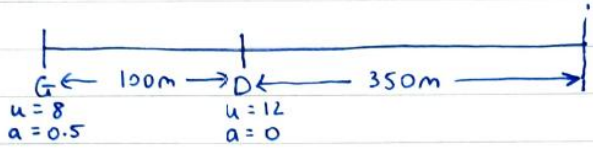
$= 112\,m$

$\Rightarrow$ Greatest Distance $= 160 - 112$

$= \boxed{48\,m}$

Q13.

i)



Let T = time to overtake

Dmitri	Gaston
$u=12 \quad a=0 \quad t=T \quad s=S_D$	$u=8 \quad a=0.5 \quad t=T \quad s=S_G$
$s = ut + \frac{1}{2}at^2$	$s = ut + \frac{1}{2}at^2$
$S_D = 12(T) + \frac{1}{2}(0)(T)^2$	$S_G = 8(T) + \frac{1}{2}(0.5)(T)^2$
$S_D = 12T$	$S_G = 8T + \frac{1}{4}T^2$

If overtaking occurs $S_G = S_D + 100$

$$\Rightarrow 8T + 0.25T^2 = 12T + 100$$

$$0.25T^2 - 4T - 100 = 0$$

$$T^2 - 16T - 400 = 0$$

$$T = \frac{16 \pm \sqrt{(-16)^2 - 4(1)(-400)}}{2(1)}$$

$$T = 29.545$$

$$\Rightarrow S_D = 12(29.54)$$

$$= 354.48 \text{ which is } > 350$$

$\Rightarrow$ Gaston does not pass before Dmitri finishes

ii) Time for Dmitri to finish?

$u=12 \quad a=0 \quad s=350 \quad t=?$

$$s = ut + \frac{1}{2}at^2$$

$$350 = 12t$$

$$\Rightarrow t = \frac{175}{6}\text{ s}$$

Time for Gaston to finish?

$$S_G = 8T + 0.25T^2$$

$$\Rightarrow S_G = 100 + 350$$

$$8T + 0.25T^2 = 450$$

$$0.25T^2 + 8T - 450 = 0$$

$$T^2 + 32T - 1800 = 0 \quad (\div 0.25)$$

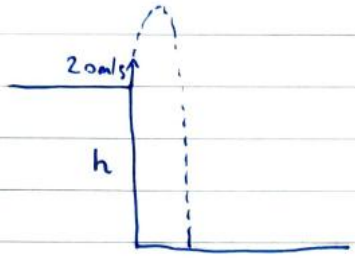
$$T = \frac{-32 \pm \sqrt{(32)^2 - 4(1)(-1800)}}{2(1)}$$

$$= 29.34\text{s} \quad \text{or} \quad \ominus \text{ time}$$

$\Rightarrow$ Gaston finishes $29.34 - \frac{175}{6}$

$= \boxed{0.17\text{s behind Dmitri}}$

Q14.



$u = 20 \quad a = -g \quad t = 5.5 \quad s = ?$

$s = ut + \frac{1}{2}at^2$

$s = 20(5.5) + \frac{1}{2}(-g)(5.5)^2$

$= -38.225$

$\Rightarrow h = \boxed{38.225\,m}$

Q15. i) $u = 0 \quad t = 20 \quad a = 0.25 \quad s = ?$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(0.25)(20)^2$

$s = \boxed{50m}$

i $v = u + at$

$v = 0 + 0.25(20)$

$v = \boxed{5m/s}$

ii) $u = 5 \quad a = g \quad s = 50 \quad t = ?$

$s = ut + \frac{1}{2}at^2$

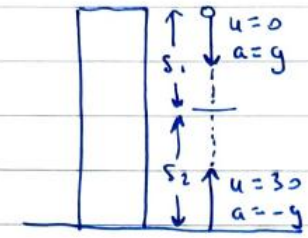
$50 = 5(t) + \frac{1}{2}(g)(t)^2$

$\Rightarrow 4.9t^2 + 5t - 50 = 0$

$t = \frac{-5 \pm \sqrt{(5)^2 - 4(4.9)(-50)}}{2(4.9)}$

$= \boxed{3.75\,s}$

Q16.



$S_1 = ut + \frac{1}{2}at^2$
$= 0(4) + \frac{1}{2}(g)(4)^2$
$= 78.4m$

$S_2 = ut + \frac{1}{2}at^2$
$= 30(4) + \frac{1}{2}(-g)(4)^2$
$= 41.6m$

$\Rightarrow h = 78.4 + 41.6$
$= \boxed{120\ m}$

Q17.

Let T = time to collide

$S_1 = ut + \frac{1}{2}at^2$
$= 25(T) + \frac{1}{2}(-g)(T)^2$
$= 25T - 4.9T^2$

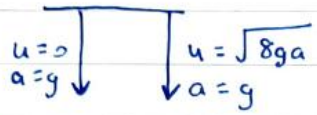
$S_2 = ut + \frac{1}{2}at^2$
$= 20(T-3) + \frac{1}{2}(-g)(T-3)^2$
$= 20T - 60 - 4.9(T^2 - 6T + 9)$
$= 20T - 60 - 4.9T^2 + 29.4T - 44.1$
$= -4.9T^2 + 49.4T - 104.1$

Collide $\Rightarrow S_1 = S_2$

$25T - 4.9T^2 = -4.9T^2 + 49.4T - 104.1$
$-25T + 49.4T = 104.1$
$24.4T = 104.1$
$T = 4.266s$

$\Rightarrow S_1 = 25(4.266) - 4.9(4.266)^2$
$= \boxed{17.5m}$

Q18.



Time for particle 1 to travel 'a':

$u = 0 \quad a = g \quad s = a \quad t = ?$

$s = ut + \frac{1}{2}at^2$

$a = 0(t) + \frac{1}{2}(g)(t)^2$

$gt^2 = 2a$

$t^2 = \frac{2a}{g}$

$\Rightarrow t = \sqrt{\frac{2a}{g}}$

Particle 2: Let T = time to collide

$u = \sqrt{8ga} \quad a = g \quad t = T - \sqrt{2a/g} \quad s = ?$

$s_2 = ut + \frac{1}{2}at^2$

$s_2 = \sqrt{8ga}\left(T - \sqrt{\frac{2a}{g}}\right) + \frac{1}{2}(g)\left(T - \sqrt{\frac{2a}{g}}\right)^2$

$= T\sqrt{8ga} - 4a + \frac{g}{2}\left(T^2 - 2\sqrt{\frac{2a}{g}} + \frac{2a}{g}\right)$

$= T\sqrt{8ga} - 4a + \frac{1}{2}gT^2 - g\sqrt{\frac{2a}{g}} + a$

Particle 1:

$u = 0 \quad t = T \quad s = ? \quad a = g$

$s_1 = ut + \frac{1}{2}at^2$

$s_1 = \frac{1}{2}gT^2$

$s_1 = s_2$

$\Rightarrow \cancel{\frac{1}{2}gT^2} = T\sqrt{8ga} - 4a + \cancel{\frac{1}{2}gT^2} - g\sqrt{\frac{2a}{g}} + a$

$T\sqrt{8ga} = 3a$

$\Rightarrow T = \frac{3a}{\sqrt{8ga}}$

$\Rightarrow s_1 = \frac{g}{2}\left(\frac{3a}{\sqrt{8ga}}\right)^2 = \frac{9a^2g}{2(8ag)} = \boxed{\frac{9a}{4}}$

2010

Q1

a) i) $u = 14 \quad v = 0 \quad s = 98$

$v^2 = u^2 + 2as$

$(0)^2 = (14)^2 + 2a(98)$

$196a = -196$

$a = -1$

$\Rightarrow$ dec = $\boxed{1 \text{ m/s}^2}$

ii) Dist in 1 sec = $14(1) = 14\text{m}$

$\Rightarrow$ 84m from lights now

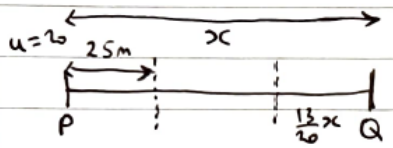
$v^2 = u^2 + 2as$

$(0)^2 = (14)^2 + 2a(84)$

$a = -7/6$

$\Rightarrow$ dec = $\boxed{7/6 \text{ m/s}^2}$

b)



In 1st second:

$u = 20 \quad s = 25 \quad t = 1$

$s = ut + \frac{1}{2}at^2$

$25 = 20(1) + \frac{1}{2}a(1)^2$

$\Rightarrow a = 10 \text{ m/s}^2$

Before last 3 secs:

$u = 20 \quad s = \frac{7}{20}x \quad a = 10$

$v^2 = u^2 + 2as$

$v^2 = (20)^2 + 2\left(\frac{7}{20}x\right)$

$v^2 = 400 + 7x$

In last 3 secs:

$u = \sqrt{400+7x} \quad a = 10 \quad s = \frac{13}{20}x \quad t = 3$

$s = ut + \frac{1}{2}at^2$

$\frac{13}{20}x = 3\sqrt{400+7x} + \frac{1}{2}(10)(3)^2$

$\frac{13x}{20} - 45 = 3\sqrt{400+7x}$

$(13x - 900)^2 = \left(60\sqrt{400+7x}\right)^2$

$169x^2 - 23400x + 81000 = 1440000 + 25200x$

$169x^2 - 48600x - 630000 = 0$

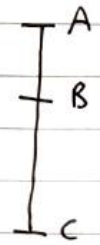
$$x = \frac{48600 \pm \sqrt{(48600)^2 - 4(169)(-630000)}}{2(169)}$$

$= \boxed{300\text{m}}$

2011

Q1

a)



A → B

$u = 0 \quad a = g \quad t = t$

$v = u + at$

$v = 0 + gt$

$v = gt$

B → C

$u = gt \quad a = g \quad t = \frac{2t}{7} \quad s = 2.45$

$s = ut + \frac{1}{2}at^2$

$2.45 = gt\left(\frac{2t}{7}\right) + \frac{1}{2}(g)\left(\frac{2t}{7}\right)^2$

$2.45 = \frac{2gt^2}{7} + \frac{2gt^2}{49}$

$2.45 = \frac{16gt^2}{49}$

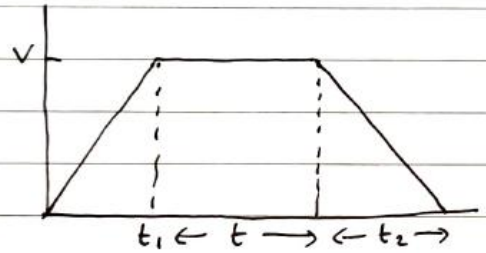
$16gt^2 = 120.05$

$t^2 = 49/64$

$t = \sqrt{49/64} = \boxed{7/8 \text{ s}}$

b)

i)



ii) Ave Speed = $\frac{\text{Tot Dist}}{\text{Tot Time}}$

Tot Dist = Area Under Curve = $\frac{1}{2}(t_1)(v) + tv + \frac{1}{2}(t_2)(v)$

$= \frac{vt_1 + 2vt + vt_2}{2}$

Tot Time = $t_1 + t_2 + t_3$

$\Rightarrow$ Ave Speed = $\frac{\frac{v(t_1 + 2t + t_2)}{2}}{t_1 + t_2 + t_3} = \frac{3v}{4}$

$\Rightarrow 2vt_1 + 4vt + 2vt_2 = 3vt_1 + 3vt_2 + 3t_3v$

$\Rightarrow t_1 + t_2 = \boxed{t}$

2015

Q1

a) Dist in 7^{th} sec $= S_7 - S_6$

$S_6 = ut + \frac{1}{2}at^2$ $\quad$ $S_7 = ut + \frac{1}{2}at^2$

$= (0)(6) + \frac{1}{2}a(6)^2$ $\quad$ $= (0)(7) + \frac{1}{2}a(7)^2$

$= 18a$ $\quad$ $= 49/2\, a$

$\Rightarrow \quad \frac{49}{2}a - 18a = 39$

$49a - 36a = 78$

$13a = 78$

$a = 6\ m/s^2$

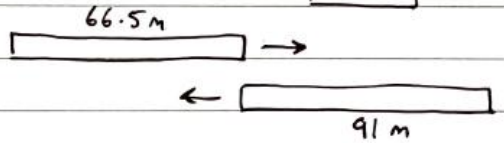
Dist in 10^{th} sec $= S_{10} - S_9$

$= (0)(10) + \frac{1}{2}(6)(10)^2 - \left[(0)(9) + \frac{1}{2}(6)(9)^2\right]$

$= 300 - 243$

$= \boxed{57m}$

b)



i) $S_1 = 18t + \frac{1}{2}\left(\frac{4}{7}\right)t^2 = 18t + \frac{2}{7}t^2$

$S_2 = 24t + \frac{1}{2}\left(\frac{8}{7}\right)t^2 = 24t + \frac{4}{7}t^2$

$S_1 + S_2 = 66.5 + 91 = 157.5$

$\Rightarrow \quad 18t + \frac{2}{7}t^2 + 24t + \frac{4}{7}t^2 = 157.5$

$126t + 2t^2 + 168t + 4t^2 = 1102.5$

$12t^2 + 588t - 2205 = 0$

$t = \boxed{3.5s}$ $\quad$ or $\ominus$

ii) After $4.5s$: $\quad S_1 = 18(4.5) + \frac{2}{7}(4.5)^2 = \frac{1215}{14}$

$S_2 = 24(4.5) + \frac{4}{7}(4.5)^2 = \frac{837}{7}$

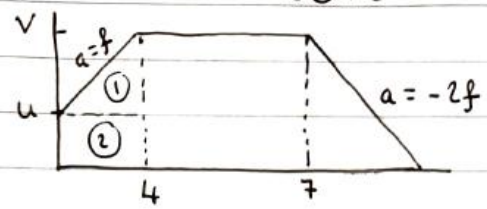
$\Rightarrow$ Dist between $= \frac{1215}{14} + \frac{837}{7} - 157.5$

$= \boxed{48.86\ m}$

2016

Q1

a) i)



ii) 45m in 3 secs

$\Rightarrow v = 45/3 = 15 m/s$

Area ① + ② = 40

$4u + \frac{1}{2}(4)(15-u) = 40$

$4u + 30 - 2u = 40$

$2u = 10$

$u = \boxed{5 m/s}$

iii) $u=5 \quad v=15 \quad t=4$

$\Rightarrow f = 5/2$

Sec 3

$u=15 \quad v=0 \quad a=-5$

$\Rightarrow s = 22.5 \quad (v^2 = u^2 + 2as)$

$\Rightarrow$ Dist = 40 + 45 + 22.5

= $\boxed{107.5m}$

b)

Particle 1

$u=u \quad a=-g \quad t=T$

$s_1 = uT - \frac{1}{2}gT^2$

Particle 2

$u=u \quad a=-g \quad t=T-2t$

$s_2 = u(T-2t) - \frac{1}{2}g(T-2t)^2$

$= uT - 2ut - \frac{1}{2}gT^2 - 2gt^2 + 2gtT$

$s_1 = s_2$

$uT - \frac{1}{2}gT^2 = uT - 2ut - \frac{1}{2}gT^2 - 2gt^2 + 2gtT$

$2ut + 2gt^2 = 2gtT$

$\Rightarrow T = \frac{u+gt}{g}$

$\Rightarrow h = u\left(\frac{u+gt}{g}\right) - \frac{g}{2}\left(\frac{u+gt}{g}\right)^2$

$= \frac{u^2+ugt}{g} - \frac{g}{2}\left(\frac{u^2+2ugt+g^2t^2}{g^2}\right)$

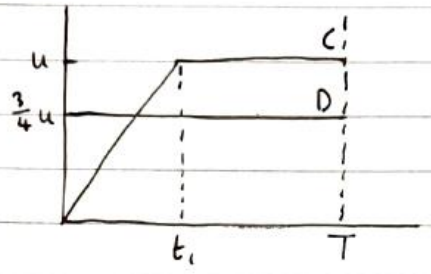
$= \frac{2u^2+2ugt}{2g} - \frac{u^2+2ugt+g^2t^2}{2g}$

$= \boxed{\frac{u^2-g^2t^2}{g}}$ Q.E.D.

2018

Q1

b) i)



Let T = time to intercept

$u=0 \quad a=a \quad v=u \quad t=t$

$v = u + at$

$u = at_1$

$\Rightarrow t_1 = u/a$

Dist travelled by C $= \frac{1}{2}\left(\frac{u}{a}\right)(u) + \left(T - \frac{u}{a}\right)(u)$

$= \frac{u^2}{2a} + uT - \frac{2u^2}{2a}$

$= uT - \frac{u^2}{2a}$

Dist travelled by D $= \frac{3}{4}uT$

$\Rightarrow uT - \frac{u^2}{2a} = \frac{3}{4}uT$

$\frac{1}{4}uT = \frac{u^2}{2a}$

$T = \frac{2u}{a}$

$\Rightarrow d = u\left(\frac{2u}{a}\right) - \frac{u^2}{2a}$

$= \frac{2u^2}{a} - \frac{u^2}{2a}$

$\Rightarrow d = \boxed{\frac{3u^2}{2a}}$

$\Rightarrow 3u^2 = 2ad$

$\Rightarrow a = \frac{3u^2}{2a}$

$\Rightarrow T = \frac{2u}{\frac{3u^2}{2d}} = \frac{2u}{1} \times \frac{2d}{3u^2} = \frac{4d}{3u}$

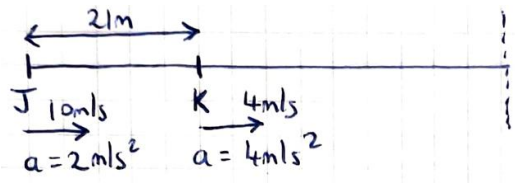
$\Rightarrow$ Time @ Speed u $= T - t_1$

$= \boxed{\frac{4d}{3u} - \frac{u}{a}}$

Q.E.D.

2024

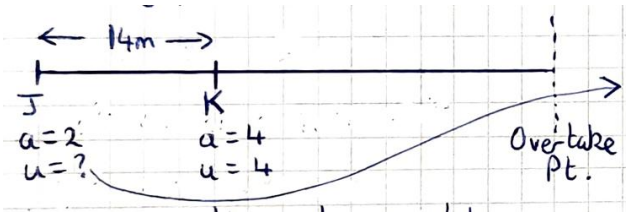
Q1(c)



In 1 second:

$$s_J = 10(1) + \frac{1}{2}(2)(1)^2 = 11m \qquad s_K = 4(1) + \frac{1}{2}(0)(1)^2 = 4m$$

=> John gains $7\ m$ on Kevin and gap reduces to $14\ m$



Speed of John after 1 sec:

$$v_J = u + at$$

$$= 10 + 2(1) = 12\ m/s$$

Let T = time to overtake

$$s_J = 12(T) + \frac{1}{2}(2)(T)^2 \qquad s_K = 4(T) + \frac{1}{2}(4)(T)^2$$

$$s_J = 12T + T^2 \qquad s_K = 4T + 2T^2$$

From 2nd diagram above: $s_K + 14 = s_J$

=> $4T + 2T^2 + 14 = 12T + T^2$

=> $T^2 - 8T + 14 = 0$

=> $T = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(14)}}{2(1)}$

=> $T = 5.41\ s$ or $T = 2.59\ s$

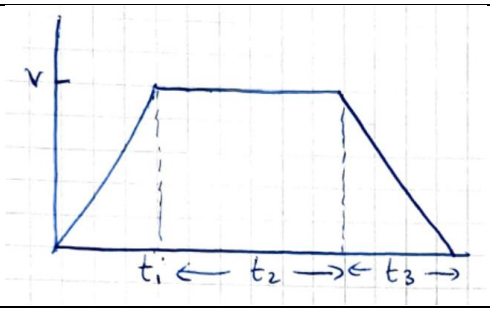
=> Ans: $T = 6.41\ s$ or $T = 3.59\ s$

Q5(b)

(b) A train departs from Connolly Station and accelerates uniformly from rest for t_1 seconds until it reaches a speed of v. The train maintains this speed for t_2 seconds, where $t_2 = 2t_1$. The train then decelerates to rest at Pearse Station in a time of t_3 seconds.

The total time taken for the journey is $T = t_1 + t_2 + t_3$.

(i) Show that $T = \frac{2d - t_2 v}{v}$, where d is the distance between Connolly Station and Pearse Station.



$t_2 = 2t_1 \Rightarrow t_1 = \frac{t_2}{2}$

$T = t_1 + t_2 + t_3$

$\Rightarrow t_3 = T - t_1 - t_2$

$\Rightarrow t_3 = T - \frac{t_2}{2} - t_2$

$\Rightarrow t_3 = T - \frac{t_2}{2} - \frac{2t_2}{2}$

$\Rightarrow t_3 = T - \frac{3t_2}{2}$

Total Distance d = Area under graph

$\Rightarrow d = \frac{1}{2}(t_1)(v) + (t_2)(v) + \frac{1}{2}(t_3)(v)$

$\Rightarrow d = \frac{1}{2}v\left(\frac{t_2}{2}\right) + vt_2 + \frac{1}{2}v(T - \frac{3t_2}{2})$

$\Rightarrow 4d = vt_2 + 4vt_2 + 2vT - 3vt_2$

$\Rightarrow 4d = 2vt_2 + 2vT$

$\Rightarrow 2d = vt_2 + vT$

$\Rightarrow vT = 2d - vt_2$

$\Rightarrow T = \frac{2d - vt_2}{v}$ Q.E.D.

(ii) If the average speed for the entire journey is $\frac{2v}{3}$, show that $T = 6t_1$.

Average Speed $= \frac{Total\ Distance}{Total\ Time} = \frac{2v}{3}$

$\Rightarrow \frac{d}{T} = \frac{2v}{3}$

$\Rightarrow 3d = 2vT$

$\Rightarrow 3d = 2v(\frac{2d - vt_2}{v})$

$\Rightarrow 3d = 4d - 2vt_2$

$\Rightarrow 2vt_2 = d$

$\Rightarrow t_2 = \frac{d}{2v}$

Subbing into expression from part (i):

$T = \frac{2d - v(\frac{d}{2v})}{v} = \frac{\frac{3d}{2}}{v} = \frac{3d}{2v}$

We know $t_1 = \frac{t_2}{2} = \frac{\frac{d}{2v}}{2} = \frac{d}{4v}$

$\Rightarrow 6t_1 = 6\left(\frac{d}{4v}\right) = \frac{3d}{2v}$

$\Rightarrow 6t_1 = T$ Q.E.D.