End of Chapter Revision Sheet: Circular Motion 6th Year

Horizontal Circular Motion Problems:

Q1. A smooth particle of mass $3\ kg$ is attached to the end of a light inextensible string of length $60\ cm$. The particle moves in a horizontal circle with constant angular velocity of $3\ rad/s$ on a smooth horizontal surface.

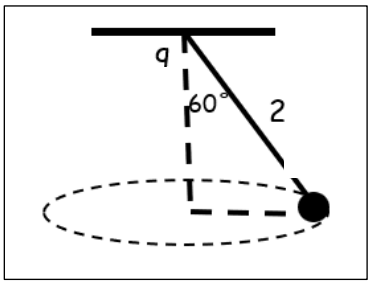
(i) Find the centripetal acceleration of the particle.

(ii) Find the tension in the string.

Q2. A smooth particle of mass $5\ kg$ is attached to a light inextensible string of length $80\ cm$. The particle describes a horizontal circle on a smooth horizontal table with a constant linear velocity of $v\ m/s$. If the maximum tension in the string is $62.5\ N$, find the maximum value of v possible without the string breaking.

Q3. A particle is placed on a playground merry-go-round, at a distance of $80\ cm$ from the centre. The coefficient of friction between the particle and the merry-go-round is 0.3. The speed of rotation of the merry-go-round is slowly increased from zero. At what angular velocity will the particle begin to slip?

Q4. A particle of mass $4\ kg$ is attached via a light inextensible string of length $2\ m$ to a fixed point q on a ceiling, as shown in the diagram on the right. The particle moves in a horizontal circle of radius r, with the centre of the circle being directly below q. The string makes an angle of $60°$ with the vertical.



(i) Find the value of r.

(ii) Find the tension in the string.

(iii) Find the angular velocity of the particle.

Vertical Circular Motion Problems:

Q5. A smooth circular hoop, of radius r, is fixed in a vertical plane. A small ring, of mass m, is threaded on the hoop and projected from the lowest point on the wire with speed u.

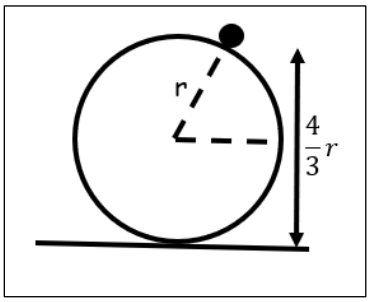
(i) If $u = \sqrt{3gr}$ find the maximum height reached by the particle.

(ii) What is the height of the particle above the lowest point when its speed $v = \sqrt{\frac{3}{2}gr}$.

(iii) Find the minimum value of u for the particle to just reach the highest point on the wire.

Q6. A body of mass m is hanging from a light inextensible string of length $2\ m$. The body is at rest with the string vertical. The body is then given a horizontal velocity of $8\ m/s$. Find the angle the string makes with the upward vertical at the instant where the string goes slack.

Q7. A particle can move on the outside surface of a smooth sphere of radius r. The sphere is fixed to a horizontal table. The particle is released from rest at a point on the outer surface of the sphere, which is $\frac{4}{3}r$ above the table, as shown in the diagram on the right. Find, in terms of r, the height of the particle above the table when it leaves the surface of the sphere.



Hooke's Law Problems:

Q8. A body of mass $0.1\ kg$ is attached to the midpoint of an elastic string pq. The string pq has elastic constant $3.5\ N/m$ and natural length $1.5\ m$. The string is stretched between two fixed points, with p being $2.5\ m$ vertically above q. Find the distance from p to the particle when the particle is in equilibrium.

Q9. An elastic string of natural length $1.5\ m$ and elastic constant $3g\ N/m$ is suspended vertically with a body of mass $3\ kg$ hanging from its end.

(i) Find the length of the string when its hanging in equilibrium.

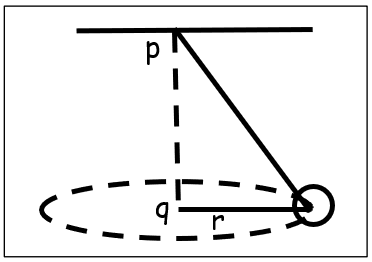
(ii) The body is pulled $0.5\ m$ below the equilibrium position and released. Find its initial upward acceleration after being released.

Q10. An elastic string of natural length $1\ m$ and elastic constant $k\ N/m$ is suspended from a ceiling with a body of mass $2\ kg$ hanging from its end. The body is pulled to the side, and it is given a velocity of $v\ m/s$ so that the system rotates as a conical pendulum. The body makes a circle of radius $0.5\ m$, and the vertical distance between the particle and ceiling is $1.2\ m$.

Show that: (i) $k = \frac{65g}{9}$ (ii) $v = \sqrt{\frac{g}{2}}$

Mixture:

Q11. The ends p and q of a light inelastic string, of length $1\ m$, are attached to two fixed points as shown in the diagram on the right. Point p is $50\ cm$ above q and a ring of mass $1\ kg$ is attached to the string at the point shown. The ring then moves in a horizontal circle with centre of q and radius r.

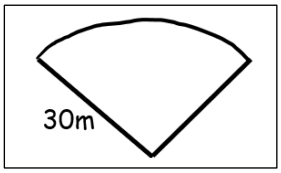


(i) Calculate the radius r of the circle.

(ii) Find the tension in the string.

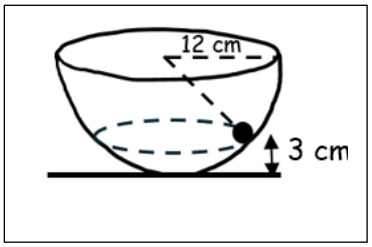
(iii) Find the angular velocity of the ring.

Q12. A humpback bridge is in the shape of a circular arc of radius $30\ m$ as shown in the diagram. What is the maximum speed the car can travel over the top of the bridge, without leaving the road?



Q13. A marble is moving on the inside surface of a smooth sphere. The radius of the sphere is $3\ m$. The marble is describing a horizontal circle of radius $\sqrt{2}\ m$. Calculate the angular velocity of the particle.

Q14. A smooth particle of mass $5\ kg$ describes a horizontal circle on the inside surface of a smooth hemisphere of radius $12\ cm$, as shown in the diagram on the right. The radius of the circle of motion is r and the centre of the circle of rotation is $3\ cm$ above the horizontal surface the sphere is on.



(i) Find the value of r.

(ii) Find the normal reaction between the particle and the hemisphere's surface.

(iii) Show that the angular velocity of the particle is $\frac{\sqrt{g}}{3}\ rad/s$.

Final challenges:

Q15. An elastic string of natural length $l\,m$ and elastic constant $k\,N/m$ is stretched between two points which are $4l\,m$ apart on the same horizontal ceiling. A particle of weight $W\,N$ is attached to the midpoint of the string and hangs in equilibrium with the two strings both inclined at $30°$ to the horizontal. Find k in terms of W and l.

Q16. A small ring is threaded on a smooth circular hoop, of radius r, which is fixed in a vertical plane. The ring is projected with speed u from the highest point of the hoop p. At the lowest point of the hoop q it collides with a second ring of equal mass, and the two rings coalesce. What is the smallest value of u, in terms of r, for which the two rings will reach point q in the subsequent motion?

Q17. A light elastic string of natural length $1.2\,m$ extends by $50\,cm$ when a small mass m is suspended from the string hanging vertically in equilibrium. If the mass is then made to describe a horizontal circle with angular velocity $\omega\,rad/s$, find an expression for the extension of the string in terms of ω.

Q18. The ends of a light string of length $2l$ are attached to two points, where one point is $2a$ vertically above the other. A smooth ring of weight W is threaded onto the string. Showa that if the ring revolved in a horizontal circle level with the lower point of attachment of the string, then its velocity is $\sqrt{\frac{(l^2-a^2)g}{a}}$ and the tension in the string is $\frac{W(l^2+a^2)}{2la}$.

Q19. A body is held at a point P on the surface of a smooth fixed sphere of radius r and centre O, where OP makes an angle of $60°$ with the upward vertical. The body is released and rolls from rest down the outside of the sphere. Find the vertical height above O of the point where the particle loses contact with the sphere and find also the horizontal distance from O of the particle when it is level with O.

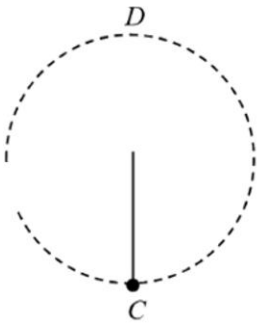
Answers:

Q1. (i) $5.4\,m/s^2$ (ii) $16.2\,N$	Q2. $\sqrt{10}\,m/s$ or $3.16\,m/s$	Q3. $\frac{7\sqrt{30}}{20}\,rads/s$ or $1.92\,rads/s$	
Q4. $\sqrt{g}\,rads/s$ or $\frac{7\sqrt{5}}{5}\,rads/s$ or $3.13\,rads/s$	Q5. (i) $\frac{3}{2}r$ (ii) $\frac{3}{4}r$ (iii) $\sqrt{5gr}\,m/s$	Q6. $65.9°$	
Q7. $\frac{11r}{9}\,m$	Q8. $1.39\,m$	Q9. (i) $2.5\,m$ (ii) $4.9\,m/s^2$	
Q11. (i) $0.375\,m$ (ii) $\frac{5g}{4}\,N$ or $12.25\,N$ (iii) $\frac{28\sqrt{15}}{15}\,rads/s$ or $7.23\,rads/s$		Q12. $7\sqrt{6}\,m/s$ or $17.15\,m/s$	
Q13. $1.92\,rads/s$	Q14. (i) $3\sqrt{7}\,m/s$ or $7.94\,cm$ (ii) $\frac{20g}{3}\,N$ or $65.33\,N$ (iii) $\frac{\sqrt{g}}{3}\,rads/s$		
Q15. $\frac{2\sqrt{3}\,W}{l(8-\sqrt{3})}$ or $\frac{0.55W}{l}$	Q16. $2gr\sqrt{3}\,m/s$	Q17. $\frac{1.2\omega^2}{2g-\omega^2}$ or $\frac{1.2\omega^2}{19.6-\omega^2}$	Q19. $\frac{r}{3}$ and $1.02r$

Past Exam Questions:

2016 Q6

6. **(a)** A small particle hanging on the end of a light inextensible string 2 m long is projected horizontally from the point C.



(i) Calculate the least speed of projection needed to ensure that the particle reaches the point D which is vertically above C.

(ii) If the speed of projection is 7 m s^{-1} find the angle that the string makes with the vertical when it goes slack.

2018 Q6

(b) A particle P is attached to one end of a light inextensible string of length d. The other end of the string is attached to a fixed point. The particle is hanging freely at rest, with the string vertical, when it is projected horizontally with speed u.
The particle moves in a complete vertical circle.

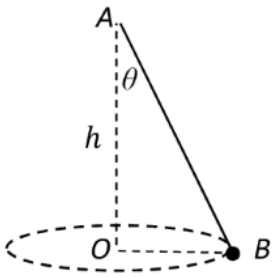
(i) Show that $u \geq \sqrt{5gd}$.

As P moves in the circle the least tension in the string is T_1 and the greatest tension is kT_1.

(ii) Given that $u = \sqrt{6gd}$, find the value of k.

2019 Q6

6. **(a)** One end *A* of a light elastic string is attached to a fixed point. The other end, *B*, of the string is attached to a particle of mass *m*. The particle moves on a smooth horizontal table in a circle with centre *O*, where *O* is vertically below *A* and $|AO| = h$. The string makes an angle θ with the downward vertical and *B* moves with constant angular speed ω about *OA*.



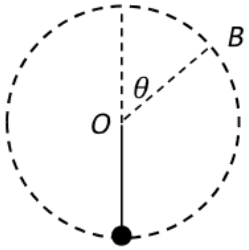
(i) Show that $\omega^2 \leq \frac{g}{h}$.

The elastic string has natural length h and elastic constant $\frac{2mg}{h}$.

(ii) Given that $\omega^2 = \frac{2g}{5h}$, find the value of θ.

2020 Q6

(b) A particle P is attached to one end of a light inextensible string of length d.
The other end of the string is attached to a fixed point *O*.
The particle is hanging freely at rest, with the string vertical, when it is projected horizontally with speed $\sqrt{3gd}$.
The particle moves in a vertical circle.
The string becomes slack when P is at the point *B*.
OB makes an angle θ with the upward vertical.

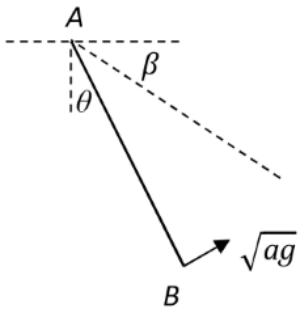


(i) Show that $\cos\theta = \frac{1}{3}$.

(ii) In terms of d, find the greatest height of P above *B* in the subsequent motion.

2017 Q6

(b) One end A of a light inextensible string of length $3a$ is attached to a fixed point. A particle of mass m is attached to the other end B of the string. The string makes an angle θ with the vertical.



The particle is held in equilibrium with the string taut and $\cos\theta = \frac{2}{3}$. The particle is then projected with speed $\sqrt{ag}$, in the direction perpendicular to AB, as show in the diagram.
In the subsequent motion the string remains taut.

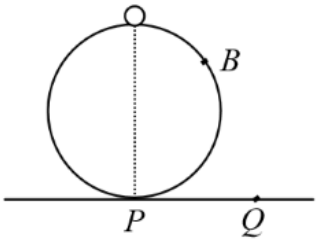
When AB makes an angle β below the horizontal, the speed of the particle is v and the tension in the string is T.

(i) Show that $v^2 = 3ag(2\sin\beta - 1)$.

(ii) Find the minimum value and the maximum value of T.

2010 Q6

6. (a) A particle of mass m kg lies on the top of a smooth sphere of radius 2 m.
The sphere is fixed on a horizontal table at P.



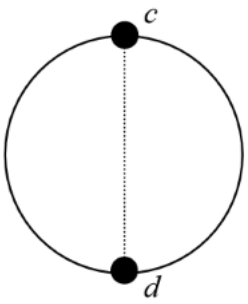
The particle is slightly displaced and slides down the sphere. The particle leaves the sphere at B and strikes the table at Q.

Find **(i)** the speed of the particle at B

(ii) the speed of the particle on striking the table at Q.

2007 Q6

(b) A bead slides on a smooth fixed circular hoop, of radius r, in a vertical plane.
The bead is projected with speed $\sqrt{10gr}$ from the highest point c.
It impinges upon and coalesces with another bead of equal mass at d.
cd is the vertical diameter of the hoop.



Show that the combined mass will not reach the point c in the subsequent motion.

Past Exam Questions:

2016: $u = 7\sqrt{2}$ and $\alpha = 80.41^0$	2018: $k = 7$	2019: 36.87^0	2020: (ii) $4d/27$
2017: $Tmin = 0.5mg, Tmax = 2mg$		2010: (i) $\sqrt{\frac{4g}{3}}$ (ii) $\sqrt{8g}$	