Topic: The Circle in Book 2 (Topics 96 to 101)

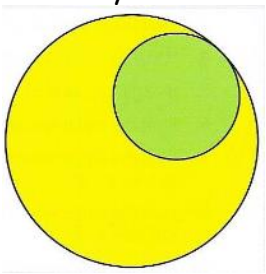
Q1. A straight gangway, with equation $x - y + 1 = 0$, lies over a circular tank with equation $x^2 + y^2 - 2x + 2y - 3 = 0$. Calculate the length of the gangway that lies over the tank. **Ans:** $\sqrt{2}$

Q2. Show that the circle $x^2 + (y - 1)^2 = 16$ lies completely inside the circle $x^2 + y^2 - 2x - 35 = 0$.

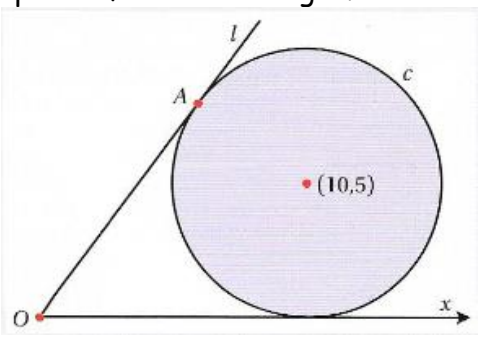
Q3. The line $2x - y + 6 = 0$ intersects the circle $x^2 + y^2 - 2y - 9 = 0$ at the points A and B. To construct an emblem, we must draw a circle with [AB] as a diameter. Find the equation of this circle.
Ans: $(x + 2)^2 + (y - 2)^2 = 5$

Q4. The circle c has centre (5, -1). The line l: $3x - 4y + 11 = 0$ is a tangent to c. (i) Show that the radius of c is 6. (ii) The line $x + py + 1 = 0$ is also a tangent to c. Find two possible values of p.
Ans: (ii) $0, -\frac{12}{35}$

Q5. In the diagram, the yellow circle represents a tray, and has equation $x^2 + y^2 - 4x - 6y + 5 = 0$. The green circle represents a plate sitting on the tray. The green circle has equation $x^2 + y^2 - 6x - 8y + 23 = 0$. Show by calculation (i) that the plate rests up against the edge of the tray (ii) that another plate of the same size could just be placed on the tray.



Q6. A disc c has centre (10, 5) and rests in a vertical plane on the x-axis as shown. A rod, l, with one end fixed to the origin, O, rests against the disc, the point of contact being A.

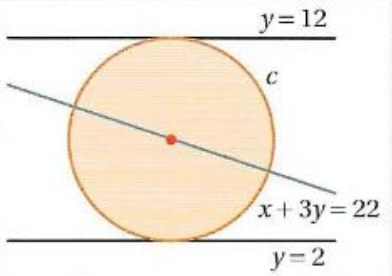


(i) Find the equation of c.
(ii) By letting the slope of l be m, and finding m, determine the equation of l.
(iii) Find the coordinates of A.
Ans: (i) $x^2 + y^2 - 20x - 10y + 100 = 0$
(ii) $4x - 3y = 0$ (iii) (6, 8)

Q7. Find the values of $k \in \mathbb{R}$ for which the line $x - y + k = 0$ is a tangent to the circle $(x - 3)^2 + (y + 4)^2 = 50$.
Ans: 3, –17

Q8. Find the equation of the circle which has its centre at the point (-2, 1) and which has the line $3x + y = 0$ as a tangent.
Ans: $(x + 2)^2 + (y - 1)^2 = \frac{5}{2}$

Q9. A disc c lies between two horizontal planks, with equations $y = 2$ and $y = 12$. A metal bar with equation $x + 3y = 22$ is clamped to the centre of the disc, holding it in place. Find the equation of the disc.



Ans: $(x - 1)^2 + (y - 7)^2 = 25$

Q10. A line containing the point p(5, 6) touches the circle $x^2 + y^2 - 4x - 4y + 4 = 0$ at k. Find |pk|.
Ans: $\sqrt{21}$

Q11. Find the equation of the circle with centre (2, 3) and which touches the x-axis.
Ans: $x^2 + y^2 - 4x - 6y + 4 = 0$.

Q12. P: $x^2 + y^2 + 2x - 2y - 23 = 0$ and Q: $x^2 + y^2 - 14x - 2y + 41 = 0$ are two circles. Prove P and Q touch externally.

Q13. Find the equations of the tangents to the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ that intersect at the point (0, 8).
Ans: $4x + 3y - 24 = 0$, $24x - 7y + 56 = 0$

Q14. Find the centre and the radius of each of the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 8x - 6y + 16 = 0$ and show that they touch externally. Write down the equation of the common tangent to these circles. **Ans:** $4x + 3y - 10 = 0$	**Q15.** Show that the line $3x + 4y - 5 = 0$ is a tangent to the circle $x^2 + y^2 - 6x - 8y + 9 = 0$.
	Q16. Find the equations of the tangents to the circle $x^2 + y^2 - 6x + 10y + 29 = 0$ which are perpendicular to the line $x - 2y + 5 = 0$. **Ans:** $2x + y + 4 = 0$ and $2x + y - 6 = 0$
Q17. Points (1, -1), (-6, -2) and (3, -5) are on a circle c. Find the equation of c. **Ans:** $x^2 + y^2 + 4x + 10y + 4 = 0$.	**Q18.** Find the equation of the chord of the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ which has (3, 5) as its midpoint. **Ans:** $x + 2y - 13 = 0$
Q19. (i) Find the equation of a circle which passes through (1, 0) and (0, 2) and which has its centre on the line $x + 3y - 11 = 0$. (ii) Prove the origin is outside the circle. **Ans:** (i) $x^2 + y^2 - 7x - 5y + 6 = 0$	**Q20.** The equation of a circle with radius length 7 is given by $x^2 + y^2 - 10kx + 6y + 60 = 0$. Find (i) the centre of the circle in terms of k and (ii) value of $k > 0$. **Ans:** (i) (5k, -3) (ii) k = 2
Q21. Find the equations of two circles which contain the points (2, 4) and (-6, 0) and that each have a radius of length $\sqrt{40}$. **Ans:** $x^2 + y^2 + 8x - 12y + 12 = 0$ and $x^2 + y^2 + 4y - 36 = 0$.	**Q22.** Find the equations of the tangents to the circle $x^2 + y^2 - 8x + 2y - 8 = 0$ and which are perpendicular to the line $3x - 4y + 1 = 0$. **Ans:** $4x + 3y + 12 = 0$, $4x + 3y - 38 = 0$
Q23. A circle has its centre in the first quadrant. The x-axis is a tangent to the circle at the point (3, 0). The y-axis cuts off a chord of length 8 units on this circle. Find the equation of the circle. **Ans:** $x^2 + y^2 - 6x - 10y + 9 = 0$	