Question 3 **(30 marks)**

(a) $ABCD$ is a parallelogram.

$|AB| = 10$ cm, $|BC| = 13$ cm, and $|\angle ABC| = 110°$.

Find the area of $ABCD$, correct to the nearest cm^2.

(b) X is an angle, with $0° \leq X \leq 360°$, and

$$\cos(2X) = \frac{\sqrt{3}}{2}$$

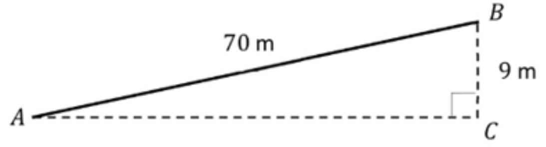
Find **all** the possible values of X.

(c) KLM is a triangle where $|MK| = 15\sqrt{3}$ cm, $|ML| = 45$ cm, and $|\angle KLM| = 25°$.
θ is the angle $\angle LKM$.

Work out the **two** possible values of θ, for $0° < \theta < 180°$.
Give each answer correct to the nearest degree.

Olga is a cyclist.

(a) The diagram below shows a road $[AB]$, which is not to scale.
AC is horizontal and BC is vertical.
$|BC| = 9$ m and $|AB| = 70$ m.



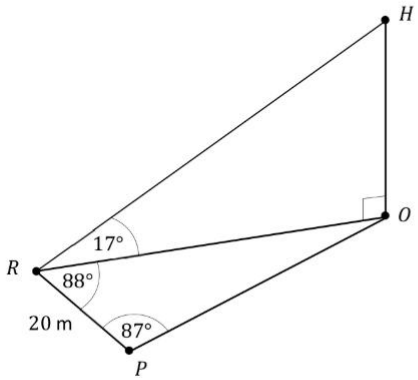
The gradient of the road $[AB]$ is $\frac{|BC|}{|AC|}$ written as a **percentage**.

Find the gradient of $[AB]$, correct to the nearest percent.

(b) Olga wants to measure the vertical height of a hill. The point H is at the top of the hill. The points R and P are 20 m apart on horizontal ground, at the bottom of the hill.

Olga measures the angle of elevation from R to H.
Taking O to be the point directly below H that is horizontal with R and P, Olga also measures the angles $\angle OPR$ and $\angle ORP$.
All of these are shown in the diagram below (not to scale).


Source: www.bikeforums.net/road-cycling



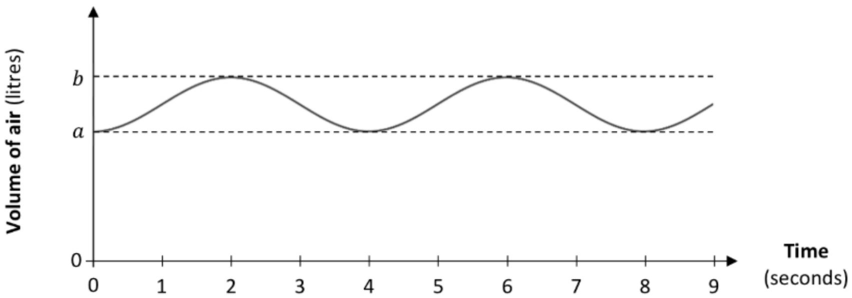
Work out the distance $|OH|$, the vertical height of the top of the hill relative to the points R and P. Give your answer correct to the nearest metre.

Olga has some tests done to measure her lung capacity.
When she is resting, the volume of air, V, in her lungs after t seconds can be modelled by:

$$V(t) = 2 - 0{\cdot}4\cos\left(\frac{\pi}{2}t\right),$$

where V is in litres, $t \geq 0$ is the time in seconds from a given point in time, and $\frac{\pi}{2}t$ is in **radians**.

The diagram below shows the graph of the function $y = V(t)$ for the first 9 seconds.



(c) Find the values marked a and b on the graph, the minimum and maximum values of V.

(d) What is the connection between $V'(t)$, the derivative of V, and whether Olga is breathing in or breathing out?

(e) Use the formula $V(t) = 2 - 0{\cdot}4\cos\left(\frac{\pi}{2}t\right)$ to find each of the following, when Olga is resting. Give each answer correct to 3 decimal places.

(i) Find the volume of air in Olga's lungs, half a second after $t = 0$.

(ii) Find the rate at which the volume of air in Olga's lungs is increasing, half a second after $t = 0$.

Question 2 **(30 marks)**

(a) Prove that $\sin(A+B) = \sin A \cos B + \cos A \sin B$.

(b) Using the formula in **part (a)**, and without using a calculator, find the value of $\sin 75°$.
Give your answer in surd form.

(c) Find all solutions of the following equation in t, for $0° \leq t \leq 360°$:

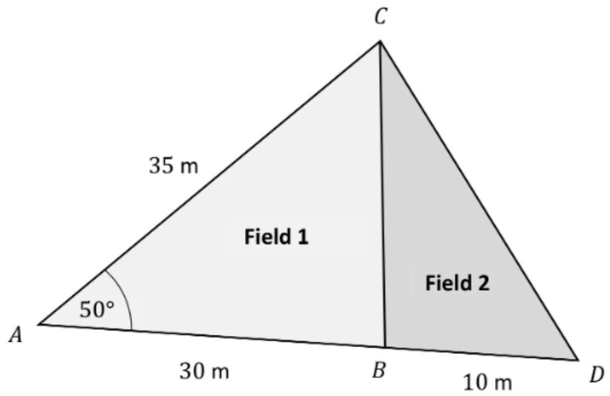
$$\sin t = \sin(2t)$$

Question 9 **(50 marks)**

Oscar is taking some measurements and is using trigonometry to work out some angles, distances, and areas.

First, Oscar takes measurements of two adjacent triangular fields, **Field 1** (ABC) and **Field 2** (BDC), as shown in the diagram below (not to scale).
B lies on the line AD. $|AB| = 30$ m, $|BD| = 10$ m, $|AC| = 35$ m, and $|\angle CAD| = 50°$.

Note: the angle ABC is **not** a right angle.



(a) Find the area of **Field 1** and, hence, find the area of **Field 2**.
Give each answer correct to the nearest m^2.

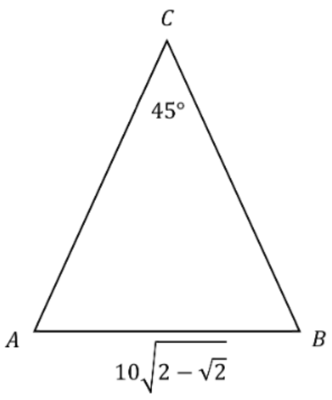
(b) Find the length of the perimeter of **Field 1**.
Give your answer correct to the nearest metre.

Question 4 **(30 marks)**

(a) **(i)** Prove that $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$.

(ii) Write $\tan 15°$ in the form $\frac{\sqrt{a}-1}{\sqrt{a}+1}$, where $a \in \mathbb{N}$.

(b) The triangle ABC is shown in the diagram below.
$|AC| = |BC|$ and $|\angle ACB| = 45°$. $|AB| = 10\sqrt{2-\sqrt{2}}$, as shown.
Find the length $|AC|$.



(b) The voltage, $V(t)$, (in Volts) of a certain alternating current is given by the function:

$$V(t) = 110\sqrt{2}\sin(120\pi t),$$

where t is in seconds.

(i) Find the period and range of the function $V(t)$.

Period: Range:

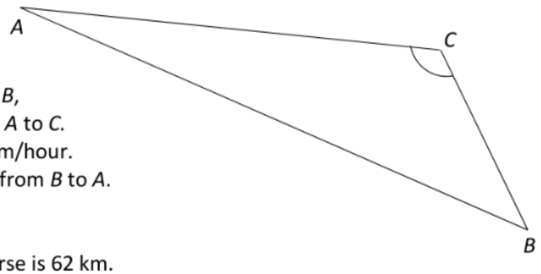
(ii) Sketch the function for $0 \leq t \leq p$, where p is the period of $V(t)$.
Indicate the period and range of the function on your graph.

(iii) Use $V(t)$ to find the voltage when $t = 6{\cdot}67$ seconds.
Give your answer correct to two decimal places.

(iv) Find one value for t where the voltage is 110 Volts.
Give your answer in the form $\frac{a}{b}$ where $a, b \in \mathbb{N}$.

Question 7 **(50 marks)**

The diagram (Triangle *ABC*) shows the 3 sections of a level triathlon course. In order to complete the triathlon, each contestant must swim 4 km from *C* to *B*, cycle from *B* to *A*, and then run 28 km from *A* to *C*. Mary can cycle at an average speed of 25 km/hour. It takes her 1 hour and 12 minutes to cycle from *B* to *A*.



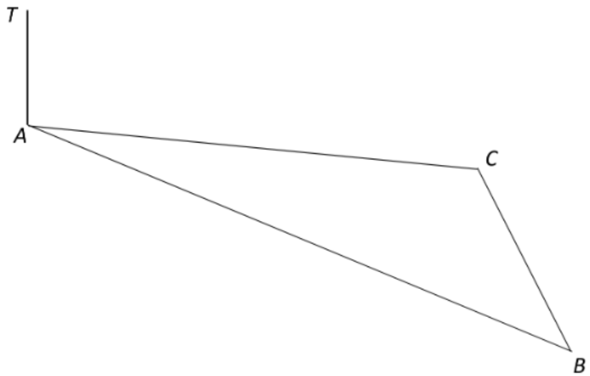
(a) Show that the **total length** of the course is 62 km.

(c) Show that $|\angle ACB| = 116{\cdot}5°$, correct to 1 decimal place.

(d) To comply with safety regulations, the region inside the triangular course must be kept clear of people. Find the area of this region. Give your answer, in km², correct to 1 decimal place.

(e) Find the shortest distance from the point *C* to the side *AB*. Give your answer in km, correct to 1 decimal place.

(f) The course is viewed from a camera tower which rises vertically from point *A*. The top of the tower is point *T*. The angle of elevation of *T* from *B* is $0{\cdot}05°$. Find $|AT|$, the vertical height of the tower. Give your answer correct to the nearest metre.



Question 4 **(30 marks)**

(a) **(i)** Prove that $\cos 2A = \cos^2 A - \sin^2 A$.

(ii) $\sin\frac{\theta}{2} = \frac{1}{\sqrt{5}}$, where $0 \leq \theta \leq \pi$.

Use the formula $\cos 2A = \cos^2 A - \sin^2 A$ to find the value of $\cos\theta$.

(b) Solve the equation:

$$\tan(B + 150°) = -\sqrt{3},$$

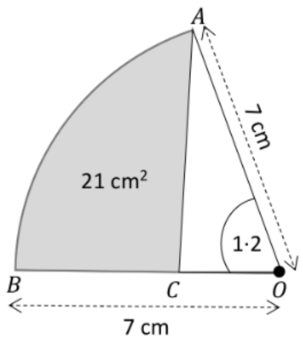
for $0° \leq B \leq 360°$.

Question 4 **(25 marks)**

(a) Find the two values of θ for which $\tan\frac{\theta}{2} = -\frac{1}{\sqrt{3}}$, where $0 \leq \theta \leq 4\pi$.

(b) The diagram shows OAB, a sector of a circle of radius 7 cm with centre O.
In the sector, $|\angle BOA| = 1{\cdot}2$ radians.
The area of the shaded region is 21 cm^2.

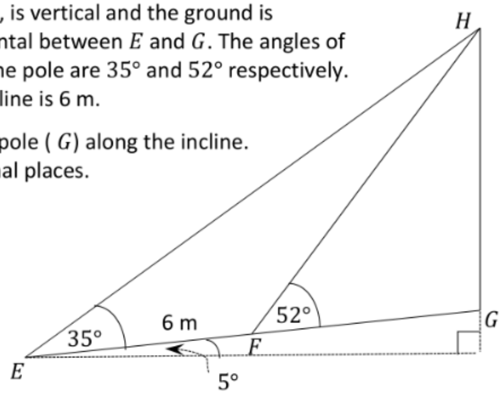
Find $|BC|$.
Give your answer correct to 1 decimal place.



Question 3 **(25 marks)**

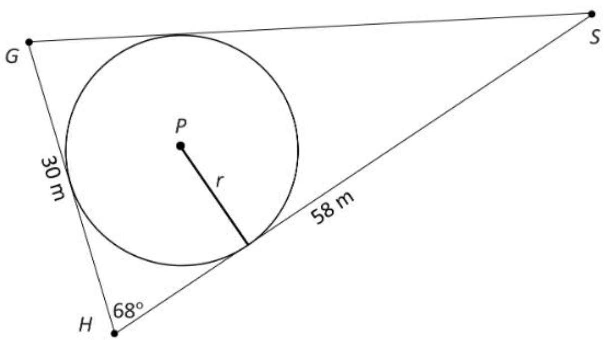
(a) A flagpole $[GH]$, shown in the diagram, is vertical and the ground is inclined at an angle of 5° to the horizontal between E and G. The angles of elevation from E and F to the top of the pole are 35° and 52° respectively. The distance from E to F along the incline is 6 m.

Find how far F is from the base of the pole (G) along the incline. Give your answer correct to two decimal places.



Question 9 **(55 marks)**

The diagram below shows a triangular patch of ground Δ*SGH*, with $|SH| = 58$ m, $|GH| = 30$ m, and $|\angle GHS| = 68°$. The circle is a helicopter pad. It is the incircle of Δ*SGH* and has centre *P*.



(a) Find $|SG|$. Give your answer in metres, correct to 1 decimal place.

(b) Find $|\angle HSG|$. Give your answer in degrees, correct to 2 decimal places.

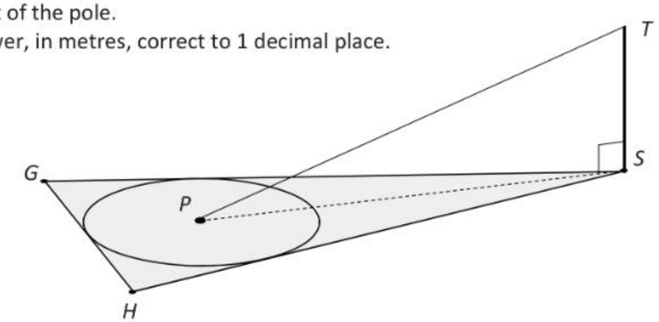
(c) Find the area of Δ*SGH*. Give your answer in m^2, correct to 2 decimal places.

(d) **(i)** Find the area of Δ*HSP*, in terms of r, where r is the radius of the helicopter pad.

(ii) Show that the area of Δ*SGH*, in terms of r, can be written as $71{\cdot}2r$ m^2.

(iii) Find the value of r. Give your answer in metres, correct to 1 decimal place.

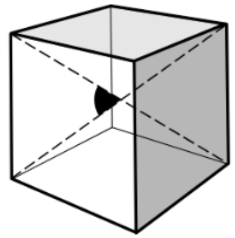
(e) $[ST]$ is a **vertical** pole at the point S.
The angle of elevation of the top of the pole from the point P is $14°$.
Find the height of the pole.
Give your answer, in metres, correct to 1 decimal place.



Question 4 **(25 marks)**

(a) Show that $\cos 2\theta = 1 - 2\sin^2\theta$.

(b) Find the cosine of the acute angle between two diagonals of a cube.



Question 4 **(25 marks)**

(a) Find **all** the values of x for which $\cos(2x) = -\frac{\sqrt{3}}{2}$, where $0° \leq x \leq 360°$.

(b) Let $\cos A = \frac{y}{2}$, where $0° < A < 90°$. Write $\sin(2A)$ in terms of y.

Question 9 **(40 marks)**

The depth of water, in metres, at a certain point in a harbour varies with the tide and can be modelled by a function of the form

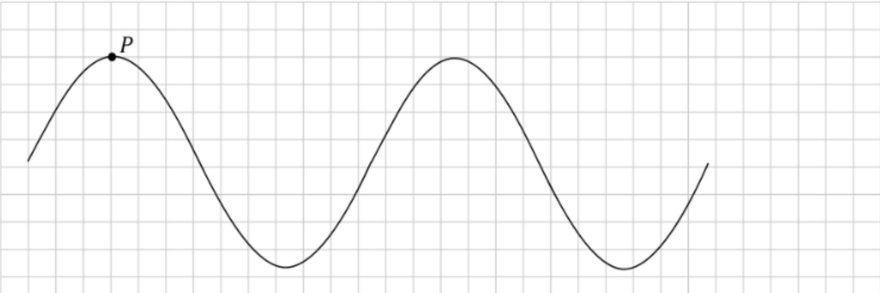
$$f(t) = a + b\cos ct$$

where t is the time in hours from the first high tide on a particular Saturday and a, b, and c are constants. (**Note**: ct is expressed in radians.)

On that Saturday, the following were noted:

- The depth of the water in the harbour at high tide was 5·5 m
- The depth of the water in the harbour at low tide was 1·7 m
- High tide occurred at 02:00 and again at 14:34.

(a) Use the information you are given to add, as accurately as you can, labelled and scaled axes to the diagram below to show the graph of f over a portion of that Saturday.
The point P should represent the depth of the water in the harbour at high tide on that Saturday morning.



(b) **(i)** Find the value of a and the value of b.

(ii) Show that $c = 0{\cdot}5$, correct to 1 decimal place.

(c) Use the equation $f(t) = a + b\cos ct$ to find the times on that Saturday **afternoon** when the depth of the water in the harbour was exactly 5·2 m.
Give each answer correct to the nearest minute.

Question 8 **(45 marks)**

The height of the water in a port was measured over a period of time. The average height was found to be 1·6 m. The height measured in metres, $h(t)$, was modelled using the function

$$h(t) = 1{\cdot}6 + 1{\cdot}5\cos\left(\frac{\pi}{6}t\right)$$

where t represents the number of hours since the last recorded high tide and $\left(\frac{\pi}{6}t\right)$ is expressed in radians.

(a) Find the period and range of $h(t)$.

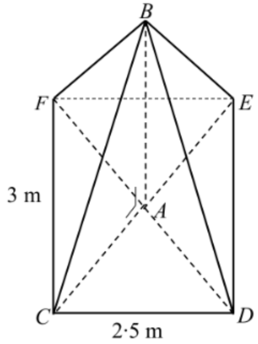
Period:

Range:

(b) Find the maximum height of the water in the port.

Question 7 **(55 marks)**

A glass Roof Lantern in the shape of a pyramid has a rectangular base $CDEF$ and its apex is at B as shown. The vertical height of the pyramid is $|AB|$, where A is the point of intersection of the diagonals of the base as shown in the diagram.
Also $|CD| = 2{\cdot}5$ m and $|CF| = 3$ m.



(a) **(i)** Show that $|AC| = 1{\cdot}95$ m, correct to two decimal places.

(ii) The angle of elevation of B from C is 50° (i.e. $|\angle BCA| = 50°$).
Show that $|AB| = 2{\cdot}3$ m, correct to one decimal place.

(iii) Find $|BC|$, correct to the nearest metre.

(iv) Find $|\angle BCD|$, correct to the nearest degree.

(v) Find the area of glass required to glaze all four triangular sides of the pyramid.
Give your answer correct to the nearest m^2.

(b) Another Roof Lantern, in the shape of a pyramid, has a square base $CDEF$. The vertical height $|AB| = 3$ m, where A is the point of intersection of the diagonals of the base as shown.
The angle of elevation of B from C is 60°
(i.e. $|\angle BCA| = 60°$).
Find the length of the side of the square base of the lantern.
Give your answer in the form $\sqrt{a}$ m, where $a \in \mathbb{N}$.