Topic 8: Probability

1) The Basics of Counting:

a) Fundamental Principle Of Counting:

If one event has **m** possible outcomes and a second event has **n** possible outcomes, then there are **m** x **n** total possible outcomes for the two events together.

e.g. 2 starters and 5 main courses

=> 10 possible dinner options

b) A Deck Of Cards:

- 52 Cards in a deck
- 4 suits: Spades (black), Clubs (black), Hearts (red) and Diamonds (red)
- Picture Cards: Jack, Queen and King in each suit (12 in total)

c) Different Strategies:

1) We can simply list all possible outcomes.
2) We can make out a **two-way table**, if there are more than two trials.
   e.g. tossing a coin two or more times
3) Sometimes it can be useful to make out a **tree diagram**, for showing all possible outcomes of two or more trials.
   e.g. chance of picking one yellow and a blue bead from a bag of 6 yellow, 5 blue

2) Permutations:

- The number of ways of rearranging n objects is given by the formula:

$$= n!$$

Example 1: Find the number of ways of rearranging the letters of the word MATHS i) with no restrictions ii) beginning with an A

i) 5 letters => 5 objects to rearrange

=> 5! = 120

ii) beginning with an 'A' means we have to fix the A in the first position and rearrange the remaining 4 letters

=> 4! = 24

Example 2: Find the number of ways of rearranging the letters of the work MARINES i) with no restrictions ii) if the vowels have to be together

i) 7 letters => 7 objects to rearrange => 7! = 5040

ii) 3 vowels have to be together

=> put 3 vowels together and treat as 1 object

=> we now have 4 consonants and 1 block of vowels to rearrange => 5 objects = 5! = 120

=> the three vowels in the vowel block can be rearranged in 3! ways

=> The total no. of Arrangements = 5! X 3! = 720

3) Basics of Probability:

a) Definition of Probability:

- The probability of an event occurring is:

$$\frac{number\ of\ successful\ outcomes}{total\ number\ of\ outcomes}$$

e.g. bag with 5 red and 4 green beads

P(Green) = $\frac{4}{9}$

Note:

- Probability values must be between 0 and 1 (see scale below)

0	¼	½	¾	1
0%	25%	50%	75%	100%
0.00	0.25	0.5	0.75	1.00
Impossible	Unlikely	Evens Chance	Likely	Certain

b) Terminology:

1. A **trial** is an act of doing an experiment in probability e.g. tossing a coin
2. An **outcome** is one of the possible results of the trial e.g. a 6 when throwing a die
3. A **sample space** is the set of all possible outcomes in a trial.
4. An **event** is the occurrence of one or more specific outcomes.
5. **Probability** is the measure of the chance of an event happening.
6. The **expected frequency** is:
   = (the no. of trials) x (relative frequency or probability)

c) Relative Frequency and Carrying Out Experiments:

- We can carry out an experiment or trials to estimate the probability of an event occurring.
  e.g. throwing a die to see how many 6's we get
- If you throw a die 20 times and a 6 comes up 3 times we could estimate the probability of throwing a 6 to be $\frac{3}{20}$.
- This estimate we get from carrying out trials, is called the **Relative Frequency**.
- The more trials that are done, the closer the relative frequency gets to the actual probability.

d) Expected Frequency:

= No. Of Trials X Relative Freq/Probability

Example 1: A die is tossed 600 times, how many times would you expect to roll a 1?

P(Throwing a 1) = $\frac{1}{6}$

=> Expected Freq of a '1' = 600 x $\frac{1}{6}$ = 100

4) Set Theory and Probability:

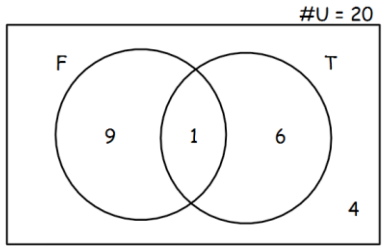
Notes:

- Sets can be used to help solve probability problems.
- Remember that A ∩ B represents A AND B whereas A ∪ B represents A OR B.

Example 1: 20 people were asked if they preferred Facebook or Twitter. 10 said Facebook, 7 said Twitter and 4 said neither. A person is selected at random from the group, what is the probability that the person selected:

i) chose Facebook and Twitter
ii) chose Facebook or Twitter
iii) chose Facebook only

- Firstly, we need to draw a Venn Diagram to represent the problem.
- 4 people chose neither => 16 people chose Facebook or Twitter
- As 10 chose Facebook and 7 chose Twitter, that means 1 person must have chosen both
- The Venn Diagram for this problem is shown on the right.



i) P(Chose Facebook AND Twitter) = $F \cap T$ = $\frac{1}{20}$

ii) P(Chose Facebook OR Twitter) = $F \cup T$ = $\frac{16}{20} = \frac{4}{5}$

iii) P(Chose Facebook Only) = $\frac{9}{20}$

5) Combined Events/Bernoulli Trials:

a) Combined Events:

Remember:

$AND = \times$

$OR = +$

Example: The probability of Paul scoring a free throw is 0.8. What is the probability of:

i) scoring three free throws in a row
ii) scoring the first and missing the next two
iii) scoring two of the three free throws

i) P(Score 1st AND Score 2nd AND Score 3rd) = 0.8 x 0.8 x 0.8 = 0.512

ii) P(Score 1st AND Miss 2nd AND Miss 3rd) = 0.8 x 0.2 x 0.2 = 0.032

iii) P(Score 1st and 2nd AND Miss 3rd) OR (Miss 1st AND Score 2nd AND 3rd) OR (Score 1st AND Miss 2nd AND Score 3rd) = (0.8 x 0.8 x 0.2) + (0.2 x 0.8 x 0.8) + (0.8 x 0.2 x 0.8) = 0.128 + 0.128 + 0.128 = 0.384

b) Bernoulli Trials:

Notes:

- When dealing with experiments whose outcomes are random and have two outcomes; success or failure.
- Trials are independent of each other i.e. the outcome of one doesn't affect the other.

Example: The probability that a person hits a target with a dart is 30%. If they throw 3 successive darts, what is the probability that the dart hits the target twice or more times?

P(Hits Twice) = (Miss 1st AND Hits 2nd AND 3rd) OR (Hit 1st AND 2nd AND Miss 3rd) OR (Hit 1st AND Miss 2nd AND Hit 3rd)
= (0.7 x 0.3 x 0.3) + (0.3 x 0.3 x 0.7) + (0.3 x 0.7 x 0.3) = 0.189

P(Hits Thrice) = Hits 1st AND Hits 2nd AND Hits 3rd
= 0.3 x 0.3 x 0.3 = 0.027

P(Hits Twice OR Thrice) = Hits Twice OR Hits Thrice
= 0.189 + 0.027
= 0.216

6) Expected Value:

- Expected Value is a way of determining if a bet is fair, good or bad.

E(X) = All Outcomes X Probability of Each Outcome

- If E(X) = 0 => Bet is **Fair**
- If E(X) > 0 => Bet is **Good**
- If E(X) < 0 => Bet is **Bad**

Example: A game costs €3 to play. If a person rolls a 2, they win €12. If they roll a 1, 3 or 5, they get their money back. If they throw a 4 or 6, they lose their money.

Probability of rolling any number is $\frac{1}{6}$

If a 2 is thrown: $\frac{1}{6} \times 12$ = €2

If a 1, 3 or 5 is thrown: $\frac{3}{6} \times 3$ = €1.50

If a 4 or 6 is thrown: = $\frac{2}{6} \times 0$ = €0

Cost of Game = €3

=> E(X) = €3.50 - €3 = €0.50

On average a player could expect to win 50cents, so this is a good bet.

If someone was using to game to raise money for a charity it would be a bad game because on average the player would win more.