Revision Sheet – Sequences & Series (Chap 8) **Review: Wed 12/12/18**

Q1. In a geometric series, the first term is 12 and the sum to infinity is 36. Find the common ratio.	**Q2.** Find the sum of the first 200 natural numbers. **Q3.** Express the recurring decimal 0.25252… in the form $\frac{p}{q}$, where $p, q \in N$.
Q4. The nth term of an arithmetic series is 3n + 2. Find S_n, the sum of the first n terms, in terms of n.	**Q5.** The sum of the first n terms of an arithmetic series is given by $S_n = 3n^2 - 4n$. Use S_n to find (i) the 1st term T_1 (ii) the sum of the 2nd term and the 3rd term $T_2 + T_3$.
Q6. Three numbers are in arithmetic sequence. Their sum is 27 and their product is 704. Find the three numbers.	**Q7.** The first 3 terms of a geometric sequence are 2x – 4, x + 1, x – 3. Find two possible values of x.
Q8. In an arithmetic series, the sum of the second term and the fifth term is 18. The 6th term is greater than the 3rd term by 9. (i) Find the 1st term and the common difference (ii) What is the smallest value of n such that $S_n > 600$?	**Q9.** (i) Find S_n, the sum of n terms of the geometric series $2 + \frac{2}{3} + \frac{2}{3^2} + \cdots + \frac{2}{3^{n-1}}$. (ii) If $S_n = \frac{242}{81}$, find the value of n. **Q10.** Write the recurring decimal 0.636363…as an infinite geometric series and hence as a fraction.
Q11. Three numbers are in geometric sequence. Their sum is 62 and their product is 1000. Find the numbers.	**Q12.** The sum to infinity of the geometric series $(4-3x) + (4-3x)^2 + (4-3x)^3 + \cdots$ is $\frac{2}{3}$. Find the value of x.
Q13. The sum of the first n terms of a series is given by $S_n = 3n^2 - 7n$. (i) Prove that the series is arithmetic (ii) Find the common difference and the first term of the series.	**Q14.** The fifth term of an arithmetic sequence is twice the second term. The two terms also differ by 9. Find the sum of the first 10 terms of the sequence.
Q15. The third term of an arithmetic sequence is 71 and the seventh term is 55. Find the first term and the common difference.	**Q16.** (i) Write an equation for the nth term of the sequence 3, 6, 12, 24, 48, …… (ii) Use logs to find the first term of this sequence to exceed one million.
Q17. The sum to infinity of a geometric series is $\frac{9}{2}$. The 2nd term of the series is -2. Find the common ratio, r, of the series.	**Q18.** In an arithmetic series, the sum of the first five terms is 50, and the sum of the next five is 125. Find the first term.
Q19. Show that the nth term of the sequence 5, 55, 555, 5555, … can be written as the sum to n terms of a geometric series and has the value $\frac{5}{9}(10^n - 1)$.	**Q20.** Given that $u_n = 2(-\frac{1}{2})^n - 2$ for all n ∈ N, (i) write down u_{n+1} and u_{n+2} (ii) show that $2u_{n+2} - u_{n+1} - u_n = 0$.
Q21. In a geometric series, the second term is 8 and the fifth term is 27. Find the first term and the common ratio.	**Q22.** u_1, u_2, u_3, u_4, u_5 ….. is a sequence where $u_1 = 2$ and $u_{n+1} = (-1)^n u_n + 3$. Evaluate u_2, u_3, u_4, u_5 and u_{10}.
Q23. a, b and c are three different, non-zero numbers. a, b, c are the first three terms of an arithmetic sequence. a, c, b are the first three terms of a geometric sequence. Find the numerical value of the common ratio of the geometric sequence.	**Q24.** The first and third terms of an arithmetic series are a and b respectively. The sum of the first n terms of this series is denoted by S_n. Find S_4 in terms of a and b. Given that S_4, S_5 and S_7 are consecutive terms of a geometric series, show that $7a^2 = 13b^2$.
Q25. If $S_n = u_1 + u_2 + \cdots + u_n$, write down an expression for S_{n-1} in terms of n. Show that $u_n = S_n - S_{n-1}$.	**Q26.** The first three terms of an arithmetic sequence are $\frac{1}{a}, \frac{1}{x}, \frac{1}{b}$. Prove that $x = \frac{2ab}{a+b}$.

Q27. A set of mirrors is arranged in such a way that light is continually reflected from one to the other. A lamp of 2000 lumens shines its light so that it reflects continuously from consecutive mirrors. Each mirror reflects $\frac{3}{5}$ of the light that hits it.

(i) Find the intensity of the light reflecting from the 10^{th} mirror.

(ii) Write an equation representing the intensity of the light reflecting from the n^{th} mirror.

(iii) After how many reflections (mirrors) will the intensity be reduced to $\frac{1}{10}$ of its original value?

Q28. A ball is dropped from a window which is 10m above horizontal ground. It strikes the ground repeatedly and each time rises to 60% of its previous height.

(i) Find the height to which it rises after the third bounce.

(ii) Find the total distance travelled by the ball before it strikes the ground for the fifth time.

(iii) Find the total distance travelled by the ball before it comes to rest.

Q29. Write the recurring decimal 5.12222….. as an infinite geometric series and hence as a fraction.

Answers:

Q1. r = $\frac{2}{3}$	**Q2.** 20,100	**Q3.** $\frac{25}{99}$	**Q4.** $\frac{n(3n+7)}{2}$	**Q5.** (i) -1 (ii) 16	**Q6.** $\frac{22}{3}, 9, \frac{32}{3}$
Q7. x = 1, 11	**Q8.** (i) a = $\frac{3}{2}$, d = 3 (ii) n = 21	**Q9.** n = 5	**Q10.** $\frac{7}{11}$		
Q11. 2, 10, 50	**Q12.** x = 1.2	**Q13.** a = -4, d = 6	**Q14.** 195	**Q15.** a = 79 d = -4	
Q16. (i) $T_n = 3.2^{n-1}$ (ii) n > 19.35 => 20^{th} term	**Q17.** $-\frac{1}{3}$	**Q18.** a = 4			
Q20. (i) $-1(-\frac{1}{2})^n - 2, \frac{1}{2}(-\frac{1}{2})^n - 2$	**Q21.** $\frac{16}{3}, \frac{3}{2}$	**Q22.** 1, 4, -1, 2, 1	**Q23.** $-\frac{1}{2}$		
Q24. a + 3b	**Q27.** (i) 12 lumens (ii) $T_n = 2000(0.6)^n$ (iii) n = 4.51 => After the 5^{th} mirror				
Q28. (i) 2.16m (ii) 36.112m (iii) 40m	**Q29.** $\frac{461}{90}$				