Topic 8: Geometry

1) The Basics:

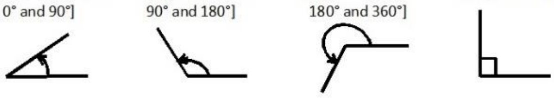
a) Terminology:

Lines:

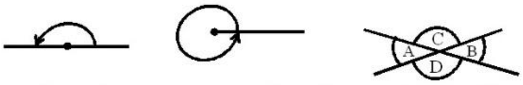
1) A Line — 2) A Half-line — 3) A Line Segment

Angles:

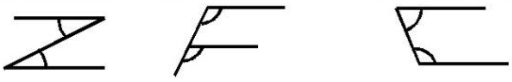
1) Acute Angle [angle between 0° and 90°]
2) Obtuse Angle [angle between 90° and 180°]
3) Reflex Angle [angle between 180° and 360°]
4) Right Angle [Angle of 90°]



5) Straight Angle [Angle of 180°]
6) Full Angle [Angle of 360°]
7) Vertically Opposite Angles ['X' Shape: A = B and C = D]

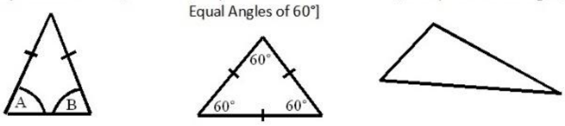


8) Alternate Angles ['Z' Shape: A = B]
9) Corresponding Angles ['F' Shape: A = B]
10) Interior Angles ['C' Shape: A + B =180°]

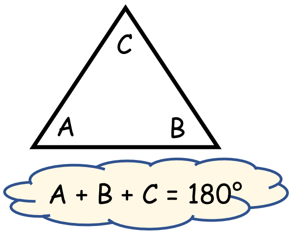


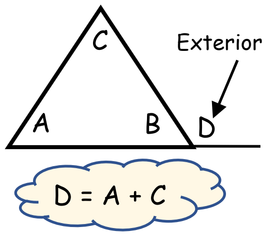
Triangles:

1) Isosceles [2 Equal Sides & A = B]
2) Equilateral [3 Equal Sides & 3 Equal Angles of 60°]
3) Scalene [No equal sides or angles]

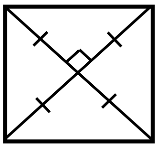


b) Properties of Triangles:

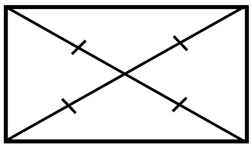




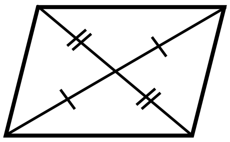
c) Properties of Quadrilaterals:



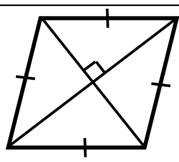
Square
- 4 equal sides
- 4 equal angles of 90°
- Opp sides are parallel
- Diagonals bisect at 90° angles



Rectangle
- Opp sides are equal & parallel
- 4 equal angles of 90°
- Diagonals bisect each other



Parallelogram
- Opp sides are equal & parallel
- Opp angles are equal
- Diagonals bisect each other

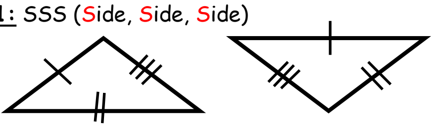


Rhombus
- 4 equal sides & opp sides are parallel
- Opp angles are equal
- Diagonals bisect at 90° angles

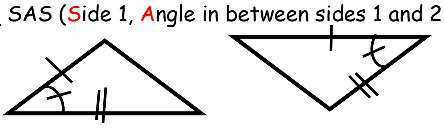
d) Conguent Triangles

- Triangles that sit exactly on top of each other
- Matching sides are called **corresponding sides**

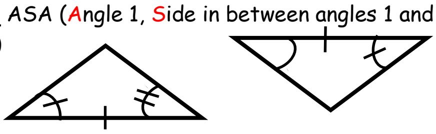
Type 1: SSS (Side, Side, Side)



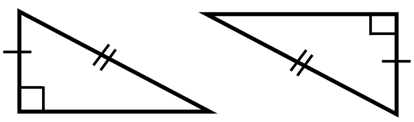
Type 2: SAS (Side 1, Angle in between sides 1 and 2, Side 2)



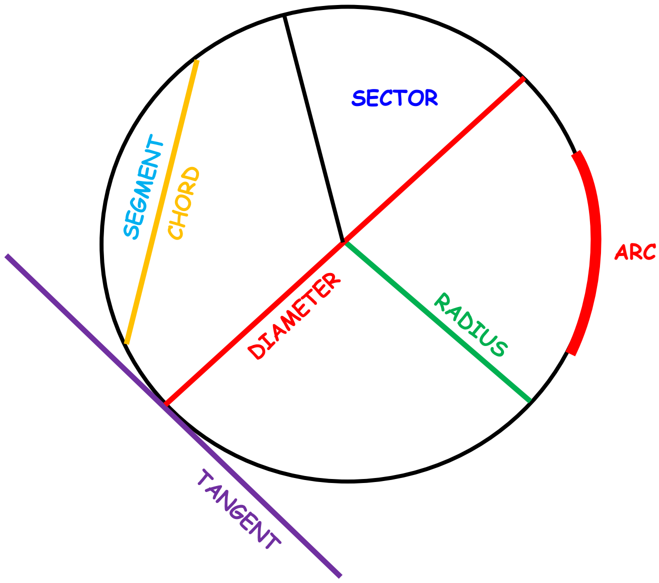
Type 3: ASA (Angle 1, Side in between angles 1 and 2, Angle 2)



Type 4: RHS (Right Angle, Hypotenuse, One other side)



e) Circle Terminology:

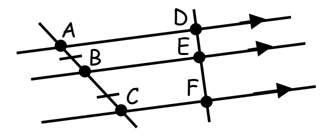


2) Theorems:

a) Theorem List:

1. Vertically opposite angles are equal in measure.
2. In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.
3. If a transversal makes equal alternate angles on two lines then the lines are parallel, (and converse).
4. The angles in any triangle add to 180°.
5. Two lines are parallel if and only if, for any transversal, the corresponding angles are equal.
6. Each exterior angle of a triangle is equal to the sum of the interior opposite angles.
7. In a parallelogram, opposite sides are equal and opposite angles are equal (and converses).
8. The diagonals of a parallelogram bisect each other.

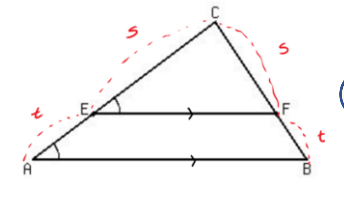
9. If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.



If |AB| = |BC|

=> |DE| = |EF|

10. Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio s:t, then it also cuts [AC] in the same ratio.

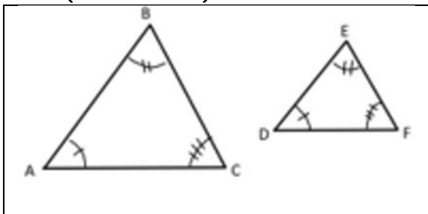


If EF is parallel to AB

=> $\frac{|AE|}{|CE|} = \frac{|BF|}{|CF|}$

or $\frac{|AE|}{|AC|} = \frac{|BF|}{|BC|}$

11. If two triangles are similar, then their sides are proportional, in order (and converse).



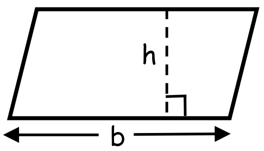
$$\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$$

OR

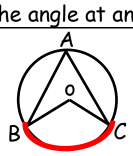
$$\frac{|DE|}{|AB|} = \frac{|EF|}{|BC|} = \frac{|DF|}{|AC|}$$

12. [Theorem of Pythagoras] In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.
13. For a triangle, base x height doesn't depend on the choice of base.
14. A diagonal of a parallelogram bisects the area.

15. The area of a parallelogram is base x height.



16. The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on that arc.



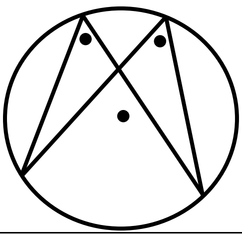
$$|< BOC| = 2(|< BAC|)$$

b) Other Theorem Terminology:

1. An **axiom** is a statement that we accept without any proof: e.g. There is exactly one line through any two given points.
3. A **theorem** is a rule that you can prove by following a certain number of logical steps or using a previous theorem or axiom. E.g. Pythagoras' Theorem
4. A **proof** is a series of logical steps that we use to show a theorem is true.
5. A **corollary** is a statement that follows readily from a previous theorem.
6. The **converse** of a statement is formed by reversing the order in which the statement is made. e.g. Statement: If P, then Q   Converse: If Q, then P.
7. '**Implies**' is a term we can use in a proof when we write down a fact or conclusion that follows from previous statements. The symbol is for implies is: =>

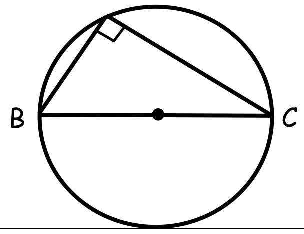
3) Corollaries: (The results below follow on from the theorems above)

1. All angles at points of a circle, standing on the same arc, are equal, (and converse).

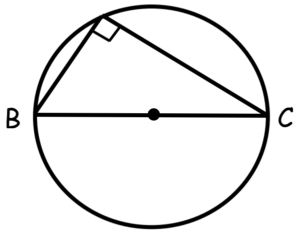


Watch for Butterfly Wings

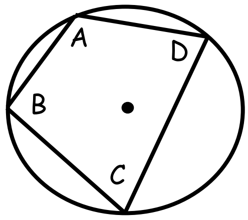
2. Each angle in a semi-circle is a right angle.



3. If the angle standing on a chord [BC] at some point, of the circle is a right-angle, then [BC] is a diameter.



4. In a cyclic quadrilateral, then opposite angles sum to 180, (and converse).



A + C = 180°

B + D= 180°

4) Constructions:

General Tips:

1. Keep your work neat and tidy.
2. Choose an appropriate pencil to draw the construction, not too dark and not too light.
3. Draw rough sketches of construction first, especially for triangles and rectangles.
4. Show all your construction lines & label your construction.

- There are 15 constructions on the course for Junior Cert Higher Level. (**See Power Points in OneNote for step by step instructions**)

Constructions List:

1. Bisector of a given angle, using only compass and straight edge.
2. Perpendicular bisector of a segment, using only compass and straight edge.
3. Line perpendicular to a given line l, passing through a given point not on l.
4. Line perpendicular to a given line l, passing through a given point on l.
5. Line parallel to a given line, through a given point.
6. Division of a line segment into 2 or 3 equal segments, without measuring it.
7. Division of a line segment into any number of equal segments, without measuring it.
8. Line segment of a given length on a given ray.
9. Angle of a given number of degrees with a given ray as one arm.

10 – 12. Triangle, given i) SSS ii) SAS or iii) ASA data

13. Right-angled triangle, given the length of the hypotenuse and one other side.

14. Right-angled triangle, given one side and one of the acute angles (several cases).

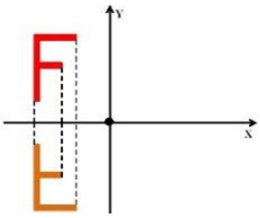
15. Rectangle, given side lengths.

5) Transformations/Symmetries:

a) Transformations:

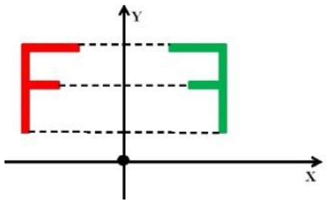
Axial Symmetry in the X-axis: (S_x)

- Shapes are mirrored/reflected in the X-axis. See example below.



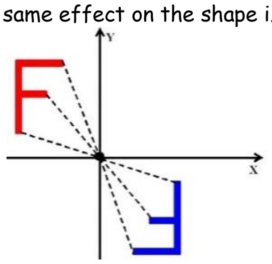
Axial Symmetry in the Y-axis: (S_y)

- Shapes are mirrored / reflected in the Y-axis. See example below.



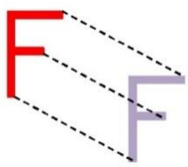
Central Symmetry in the Origin: (S_o)

- Shapes end up flipped and rotated as shown below.
- Central symmetry in a point other than the origin would have the same effect on the shape i.e. flipped and rotated



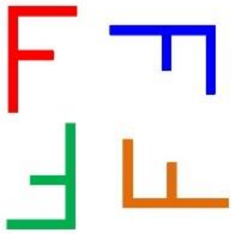
Translation:

- Note that shapes don't change when translated as the shape just '**slides**' to another position



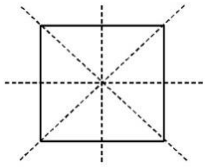
Rotations:

- The shape in blue below is a rotation of the red shape 90° clockwise. The green is a rotation of 180°. Note that it looks similar to the central symmetry in a point image from above. The orange is a rotation of 270° clockwise.

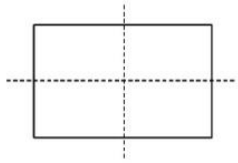


b) Axes of Symmetries of Shapes:

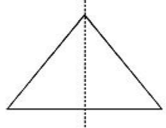
Square:
A square has 4 axes of symmetry, as shown below.



Rectangle:
A rectangle has 2 axes of symmetry, as shown below.



Triangle:
An isosceles triangle has 1 axis of symmetry, as shown below. If the triangle was an equilateral triangle it would have two more axes of symmetry from the other two vertices of the triangle.



Circle:
A circle has an infinite number of axes of symmetry, as shown below.