

Topic 7: Probability

1) The Basics of Counting:

<p>a) Fundamental Principle Of Counting: If one event has m possible outcomes and a second event has n possible outcomes, then there are $m \times n$ total possible outcomes for the two events together. e.g. 2 starters and 5 main courses \Rightarrow 10 possible dinner options</p>	<p>c) Different Strategies: 1) We can simply list all possible outcomes. 2) We can make out a two-way table, if there are more than two trials. e.g. tossing a coin two or more times 3) Sometimes it can be useful to make out a tree diagram, for showing all possible outcomes of two or more trials. e.g. chance of picking one yellow and a blue bead from a bag of 6 yellow, 5 blue</p>
<p>b) A Deck Of Cards:</p> <ul style="list-style-type: none"> 52 Cards in a deck 4 suits: Spades & Clubs (black), Hearts & Diamonds (red) Picture Cards: Jack, Queen and King in each suit (12 in total) 	

2) Basics of Probability:

<p>a) Definition of Probability:</p> <ul style="list-style-type: none"> The probability of an event occurring is: <div style="text-align: center; border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> $\frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$ </div> <p>e.g. bag with 5 red and 4 green beads $P(\text{Green}) = \frac{4}{9}$</p> <p>Note:</p> <ul style="list-style-type: none"> Probability values must be between 0 and 1 (see scale below) <div style="text-align: center;"> </div>	<p>b) Terminology:</p> <ol style="list-style-type: none"> Trial: doing an experiment in probability e.g. tossing a coin Outcome: one of the possible results of the trial e.g. a 6 when throwing a die Sample space is the set of all possible outcomes in a trial. Event is the occurrence of one or more specific outcomes. Probability is the measure of the chance of an event happening. <p>c) Relative Frequency and Carrying Out Experiments:</p> <ul style="list-style-type: none"> We can carry out an experiment or trials to estimate the probability of an event occurring. e.g. throwing a die to see how many 6's we get If you throw a die 20 times and a 6 comes up 3 times we could estimate the probability of throwing a 6 to be $\frac{3}{20}$. This estimate we get from carrying out trials, is called the Relative Frequency. More trials are done \Rightarrow closer the rel freq and probability.
-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

3) Set Theory and Probability:

<p>Notes:</p> <ul style="list-style-type: none"> Sets can be used to help solve probability problems. Remember that $A \cap B$ represents A AND B whereas $A \cup B$ represents A OR B. <p>Example 1: 20 people asked if they preferred Facebook or Twitter. 10 said Facebook, 7 said Twitter and 4 said neither. Person selected at random from the group...what is the probability that the person selected:</p> <ol style="list-style-type: none"> chose Facebook and Twitter chose Facebook or Twitter chose Facebook only? <ul style="list-style-type: none"> Firstly, we need to draw a Venn Diagram to represent the problem. 4 people chose neither \Rightarrow 16 people chose Facebook or Twitter As 10 chose Facebook and 7 chose Twitter \Rightarrow 1 person chose both The Venn Diagram for this problem is shown on the right. 	<div style="text-align: right; margin-bottom: 5px;">#U = 20</div> <p>i) $P(\text{Chose Facebook AND Twitter}) = F \cap T = \frac{1}{20}$ ii) $P(\text{Chose Facebook OR Twitter}) = F \cup T = \frac{16}{20} = \frac{4}{5}$ iii) $P(\text{Chose Facebook Only}) = \frac{9}{20}$</p>
------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

4) Combined Events/Bernoulli Trials:

<p>Remember:</p> <div style="text-align: center; border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> $\text{AND} = \times$ $\text{OR} = +$ </div> <p>Example 1: Probability of Paul scoring a free throw is 0.8. What is the probability of:</p> <ol style="list-style-type: none"> scoring three free throws in a row scoring the first and missing the next two scoring two of the three free throws? <p>i) $P(\text{Score 1}^{\text{st}} \text{ AND Score 2}^{\text{nd}} \text{ AND Score 3}^{\text{rd}}) = 0.8 \times 0.8 \times 0.8 = 0.512$ ii) $P(\text{Score 1}^{\text{st}} \text{ AND Miss 2}^{\text{nd}} \text{ AND Miss 3}^{\text{rd}}) = 0.8 \times 0.2 \times 0.2 = 0.032$ iii) $P(\text{Score 1}^{\text{st}} \text{ and 2}^{\text{nd}} \text{ AND Miss 3}^{\text{rd}}) \text{ OR } (\text{Miss 1}^{\text{st}} \text{ AND Score 2}^{\text{nd}} \text{ AND Score 3}^{\text{rd}}) \text{ OR } (\text{Score 1}^{\text{st}} \text{ AND Miss 2}^{\text{nd}} \text{ AND Score 3}^{\text{rd}})$ $= (0.8 \times 0.8 \times 0.2) + (0.2 \times 0.8 \times 0.8) + (0.8 \times 0.2 \times 0.8) = 0.128 + 0.128 + 0.128 = 0.384$</p>	
--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	--