Sample Paper 3 Marking Scheme

Question 1

(a)

(i) $x^2 - 7x + 10 = 0$ (5)

$x = 2 \text{ OR } x = 5$

$u_n = l(2)^n + m(5)^n$ (5)

$593 = l(2)^0 + m(5)^0,\ 1165 = l(2)^1 + m(5)^1$

$l = 600,\ m = -7$

$u_n = 600(2)^n - 7(5)^n$ (5)

(ii) $600(2)^n - 7(5)^n < 0$

$n > 4.86$

$\therefore n = 5$ (5)

(b)

Two cars, A and B, travel along a straight level road in opposite directions. A passes point *P* with speed 4 m s^{-1} and uniform acceleration 2 m s^{-2}. Three seconds later B passes point *Q* with speed 5 m s^{-1} and uniform acceleration 4 m s^{-2}.

The distance from *P* to *Q* is 1143 m.

The cars meet *t* seconds after A passes *P*.

(i) Find the value of *t*.

(ii) Find the distance from *P* to the meeting point.

(iii) Find the distance between the cars when A is 160 m from the meeting point, before the cars meet.

(i)

$$S_A = 4t + \frac{1}{2}(2)t^2 \qquad (5)$$

$$S_B = 5(t-3) + \frac{1}{2}(4)(t-3)^2 \qquad (5)$$

$$S_A + S_B = 1143$$

$$4t + t^2 + 5(t-3) + 2(t-3)^2 = 1143$$

$$t = 20 \qquad (5)$$

(ii)

$$S_A = 4(20) + \frac{1}{2}(2)(20)^2$$

$$S_A = 480 \qquad (5)$$

(iii)

$$S_A = 4t + \frac{1}{2}(2)t^2 = 480 - 160$$

$$t^2 + 4t - 320 = 0$$

$$t = 16 \qquad (5)$$

$$S_B = 5(16-3) + \frac{1}{2}(4)(16-3)^2 = 403$$

$$d = 1143 - 320 - 403 = 420 \text{ m} \qquad (5) \qquad (30)$$

Question 2
(a)

(i) $20 - \frac{1}{5}(0)^2 = 20$ m/s (5)

(ii) $a(t) = \frac{dv(t)}{dt} = -\frac{2}{5}t$ (5)

$a(4) = -\frac{2}{5}(4) = -1.6$

Deceleration $= 1.6$ m/s^2 (5)

(iii) $0 = 20 - \frac{1}{5}t^2$

$t = 10$ s (5)

(iv) $\int_0^{10} v(t) = 20t - \frac{1}{15}t^3 \Big|_0^{10} = \frac{400}{3}$ m (5) + (5)

(b)

(i)

Stage	State	Action	Destination	Value
1	G	GT	T	20*
	H	HT	T	24*
2	D	DG	G	38*
	E	EG	G	35*
		EH	H	38
	F	FH	H	40
		FG	G	36*
3	A	AD	D	60*
		AE	E	60*
	B	BD	D	65
		BE	E	63
		BF	F	57*
	C	CE	E	58*
		CF	F	60
4	S	SA	A	88
		SB	B	84*
		SC	C	88

(5)

(5)

Optimal Solution: SBFGT

(5)

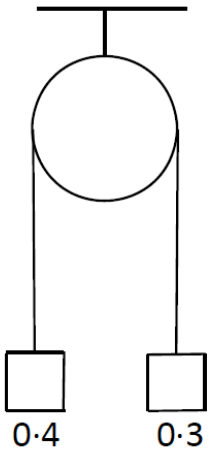
(ii) $\frac{84000}{4} = €21000$

(5)

Question 3

(a)

Two particles of masses 0·4 kg and 0·3 kg are attached to the ends of a light inextensible string which passes over a light smooth fixed pulley. They are held at the same level, as shown in the diagram.



The system is released from rest.

Find

(i) the tension in the string

(ii) the speed of the 0·4 kg mass when it has descended 0·7 m.

(i) $0.4g - T = 0.4a$ (5)

$T - 0.3g = 0.3a$ (5)

$a = \frac{g}{7} = 1.4$ (5)

$T = 0.3g + 0.3a$

$T = 3.36$ N (5)

(ii) $v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times 1.4 \times 0.7$

$v = 1.4 \text{ m s}^{-1}$ (5) (25)

(b)

A particle of mass *m* is suspended vertically from a fixed point *O* by a light inelastic string of length *d* metres.

The particle is projected horizontally with speed *u*, where $u^2 = 4gd$.

Show the string goes slack when it makes an angle $\cos^{-1}\frac{2}{3}$ with the upward vertical through *O*.

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgh \qquad (5)$$

$$4gd = v^2 + 2g(d + d\cos\alpha)$$

$$v^2 = 2gd - 2gd\cos\alpha \qquad (5)$$

$$T + mg\cos\alpha = \frac{mv^2}{d} \qquad (5)$$

$$0 + mg\cos\alpha = \frac{mv^2}{d} \qquad (5)$$

$$gd\cos\alpha = 2gd - 2gd\cos\alpha$$

$$3\cos\alpha = 2$$

$$\alpha = \cos^{-1}\left(\frac{2}{3}\right) \qquad (5) \qquad (25)$$

Question 4

(a)

A particle is projected from a point *P* with speed u m s^{-1} at an angle α to the horizontal.

(i) Show that the range of the particle is $\frac{2u^2 \sin\alpha \cos\alpha}{g}$.

The particle is 24·5 m above the horizontal ground after 5 seconds and it strikes the ground 235·2 m from *P*.

(ii) Find the value of u.

(i)

$$r_j = 0$$

$$u\sin\alpha \times t - \frac{1}{2}gt^2 = 0$$

$$t = \frac{2u\sin\alpha}{g}$$ (5)

$$\text{Range} = u\cos\alpha \times t$$

$$= u\cos\alpha \times \frac{2u\sin\alpha}{g}$$

$$= \frac{2u^2\sin\alpha\cos\alpha}{g}$$ (5)

(ii)

$$r_j = 24.5$$

$$u\sin\alpha \times 5 - 4.9 \times 5^2 = 24.5$$

$$u\sin\alpha = 29.4$$ (5)

$$\text{Range} = u\cos\alpha \times \frac{2u\sin\alpha}{g} = 235.2$$

$$u\cos\alpha \times \frac{58.8}{g} = 235.2$$

$$u\cos\alpha = 39.2$$ (5)

$$(u\sin\alpha)^2 + (u\cos\alpha)^2 = 29.4^2 + 39.2^2$$

$$u^2(\sin^2\alpha + \cos^2\alpha) = 2401$$

$$u^2 = 2401$$

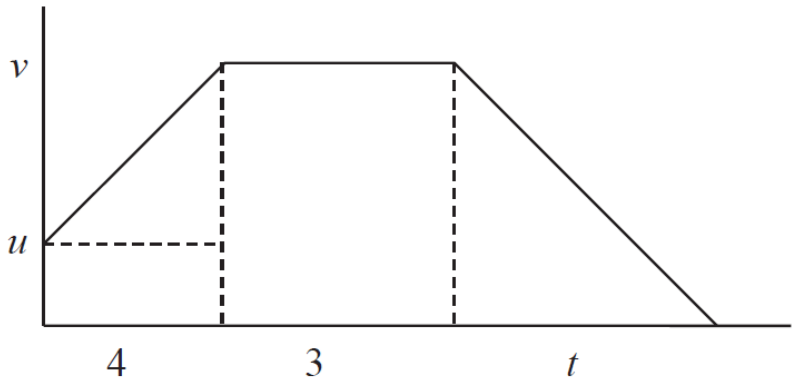
$$u = 49 \text{ m s}^{-1}$$ (5) (25)

(b)

A car has an initial speed of u m s^{-1}. It moves in a straight line with constant acceleration f for 4 seconds. It travels 40 m while accelerating.
The car then moves with uniform speed and travels 45 m in 3 seconds.
It is then brought to rest by a constant retardation $2f$.

(i) Draw a speed-time graph for the motion.
(ii) Find the value of u.
(iii) Find the total distance travelled.

(i)



5

(ii) $3v = 45$

$v = 15$

5

$4u + \frac{1}{2}(4)(15-u) = 40$

$4u + 30 - 2u = 40$

$u = 5 \text{ m s}^{-1}$

5

(iii) $f = \dfrac{15-5}{4} = 2.5$

5

$t = \dfrac{15}{2f} = 3$

$d = 40 + 45 + \frac{1}{2}(3)15$

$= 107.5 \text{ m}$

5

25

Question 5

(a)

If $\frac{dy}{dx} = 3\sin 3x + \cos 5x$ and $y = 1$ when $x = \frac{\pi}{4}$, find the value of y when $x = \frac{\pi}{2}$.
Give your answer correct to 2 decimal places.

$$\frac{dy}{dx} = 3\sin 3x + \cos 5x$$

$$\int dy = \int (3\sin 3x + \cos 5x)\, dx \qquad (5)$$

$$[y]_1^y = \left[-\cos 3x + \frac{\sin 5x}{5}\right]_{\pi/4}^{\pi/2} \qquad (5), (5)$$

$$y - 1 = \left[0 + \frac{1}{5}\right] - \left[\frac{1}{\sqrt{2}} - \frac{1}{5\sqrt{2}}\right] \qquad (5)$$

$$y = 0.63 \qquad (5) \qquad (25)$$

(b)

a) $u_n = 2u_{n-1} + 2(3^n)$

$u_n - 2u_{n-1} = 2(3)^n$

Char Eqn: $x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0 \quad \text{or} \quad x = 2$

$\Rightarrow u_n = l(0)^n + m(2)^n$ (5)

Part Soln: $u_n = a(3)^n + b$

$u_{n-1} = a(3)^{n-1} + b$

$= \frac{a(3)^n}{3} + b$

$\Rightarrow a(3)^n + b - 2\left(\frac{a(3)^n}{3} + b\right) = 2(3)^n$ (5)

$\frac{3a(3)^n}{3} + b - \frac{2a(3)^n}{3} - 2b = 2(3)^n$

$\frac{a}{3}(3)^n - b = 2(3)^n$

$\Rightarrow \frac{a}{3} = 2 \qquad \boxed{b = 0}$

$\boxed{a = 6}$ (5)

$\Rightarrow$ Combined Soln: $u_n = m(2)^n + 6(3)^n$

$u_0 = 1 \Rightarrow m(2)^0 + 6(3)^0 = 1$

$m + 6 = 1$

$m = -5$

$\Rightarrow u_n = \boxed{-5(2)^n + 6(3)^n}$ (5)

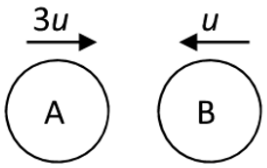
b) $u_8 = -5(2)^8 + 6(3)^8$

$= \boxed{38086}$ (5)

Question 6

(a)

A smooth sphere A of mass m, moving with speed $3u$ on a smooth horizontal table collides directly with a smooth sphere B of mass $2m$, moving in the opposite direction with speed u.
The directions of motion of A and B are reversed by the collision.

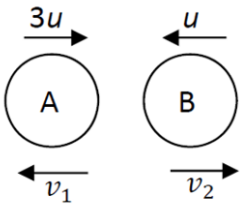


The coefficient of restitution between A and B is e.

(i) Find the speed, in terms of u and e, of each sphere after the collision.

Subsequently B hits a wall at right angles to the line of motion of A and B.
The coefficient of restitution between B and the wall is $\frac{1}{2}$.
After B rebounds from the wall there is a further collision between A and B.

(ii) Show that $\frac{1}{8} < e < \frac{1}{4}$.



(i) PCM $\quad m(3u) + 2m(-u) = m(-v_1) + 2mv_2$ (5)

NEL $\quad -v_1 - v_2 = -e(3u + u)$ (5)

$$-v_1 + 2v_2 = u$$

$$v_1 + v_2 = 4eu$$

$$v_1 = \frac{u(8e-1)}{3} \qquad v_2 = \frac{u(1+4e)}{3}$$ (5), (5)

(ii) $\frac{1}{2}v_2 = \frac{u(1+4e)}{6}$

2nd collision if $\frac{1}{2}v_2 > v_1$

$$\frac{u(1+4e)}{6} > \frac{u(8e-1)}{3}$$

$$1 + 4e > 16e - 2$$

$$e < \frac{1}{4} \qquad (5)$$

$$v_1 > 0$$

$$\frac{u(8e-1)}{3} > 0$$

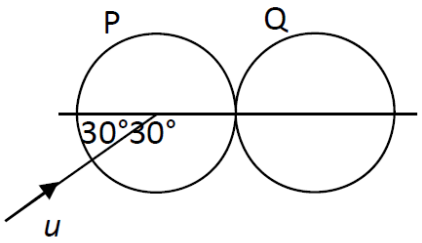
$$8e - 1 > 0$$

$$e > \frac{1}{8}$$

$$\frac{1}{8} < e < \frac{1}{4} \qquad (5) \qquad (30)$$

(b)

A smooth sphere P has mass m_1 and speed u.
It collides obliquely with a smooth sphere Q, of mass m_2, which is at rest.
Before the collision the direction of P makes an angle of 30° to the line of centres, as shown in the diagram.



The coefficient of restitution between the spheres is e.

Prove that P will turn through a right-angle if $4m_1 = (3e - 1)m_2$.

P	m_1	$\frac{u\sqrt{3}}{2}\ \vec{i} + \frac{u}{2}\ \vec{j}$	$v_1\ \vec{i} + \frac{u}{2}\ \vec{j}$
Q	m_2	$0\ \vec{i} + 0\ \vec{j}$	$v_2\ \vec{i} + 0\ \vec{j}$

PCM $\quad m_1\left(\frac{u\sqrt{3}}{2}\right) + m_2(0) = m_1 v_1 + m_2 v_2$ (5)

NEL $\quad v_1 - v_2 = -e\left(\frac{u\sqrt{3}}{2} - 0\right)$ (5)

$$\frac{m_1}{m_2} v_1 + v_2 = \frac{m_1}{m_2}\left(\frac{u\sqrt{3}}{2}\right)$$

$$v_1 - v_2 = -e\left(\frac{u\sqrt{3}}{2}\right)$$

$$\frac{m_1}{m_2} v_1 + v_1 = \frac{m_1}{m_2}\left(\frac{u\sqrt{3}}{2}\right) - e\left(\frac{u\sqrt{3}}{2}\right) \qquad (5)$$

$$\tan 60 = \frac{\frac{u}{2}}{-v_1}$$

$$v_1 = -\frac{u}{2\sqrt{3}} = -\frac{u\sqrt{3}}{6}$$

$$\frac{m_1}{m_2}\left(-\frac{u}{2\sqrt{3}}\right) - \frac{u}{2\sqrt{3}} = \frac{m_1}{m_2}\left(\frac{u\sqrt{3}}{2}\right) - e\left(\frac{u\sqrt{3}}{2}\right)$$

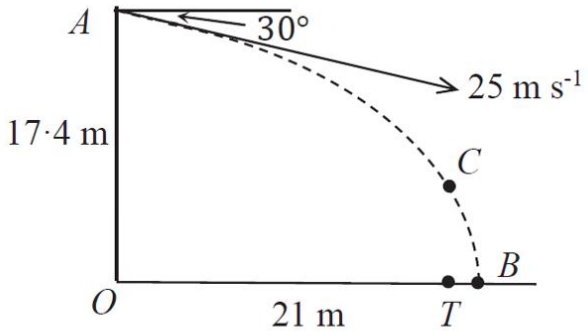
$$\frac{m_1}{m_2} + 1 = -\frac{m_1}{m_2}(3) + \mathrm{e}(3)$$

$$4m_1 = (3e - 1)m_2 \qquad (5) \qquad (20)$$

Question 7

(a)

A ball is thrown from a point A at a target T, which is on horizontal ground. The point A is 17·4 m vertically above the point O on the ground. The ball is thrown from A with speed 25 m s^{-1} at an angle of 30° below the horizontal. The distance OT is 21 m.
The ball misses the target and hits the ground at the point B, as shown in the diagram.
Find



(i) the time taken for the ball to travel from A to B

(ii) the distance TB.

The point C is on the path of the ball vertically above T.

(iii) Find the speed of the ball at C.

(i) $17.4 = 25\sin 30.t + \frac{1}{2}gt^2$ — 5

$17.4 = 12.5t + 4.9t^2$

$t = 1$ — 5

(ii) $|OB| = 25\cos 30.(1) = 21.65$

$|TB| = 21.65 - 21$

$= 0.65$ m — 5

(iii) $25\cos 30 \times t = 21$

$t = 0.97$ — 5

$v_{\bar{i}} = 25\cos 30 = 21.65$

$v_{\bar{j}} = 25\sin 30 + g(0.97) = 22.01$

$v = \sqrt{21.65^2 + 22.01^2}$

$= 30.87$ m s^{-1} — 5

25

(b)

P, the population of insects in a region, grows at a rate that is proportional to the current population.

$$\frac{dP}{dt} = kP$$

where k is a positive constant. In the absence of any outside factors the population will triple in 15 days.

(i) Find the value of k.

A scientist begins to remove 10 insects from the population each day.

(ii) If there are initially 120 insects in the region the population will not survive. After how many days will the population die out?

(i)

$\frac{dP}{dt} = kP$

$\int \frac{dP}{P} = k \int dt$ (5)

$[\ln P]_P^{3P} = k[t]_0^{15}$ (5)

$ln3P - lnP = 15k$

$k = \frac{ln3}{15} = 0.07324$ (5)

(ii)

$\frac{dP}{dt} = \frac{ln3}{15}P - 10$

$\int \frac{dP}{\frac{ln3}{15}P - 10} = \int dt$

$\frac{15}{ln3}\left[ln\left|\frac{ln3}{15}P - 10\right|\right]_{120}^{0} = [t]_0^t$ (5)

$t = \frac{15}{ln3}(ln10 - ln1.2111)$

$t = 28{\cdot}8$ days (5) (25)

Question 8

(a)

Speed of the joint mass after impact

$$\frac{1}{2}MV_1{}^2 + Mgh_1 = \frac{1}{2}MV_2{}^2 + Mgh_2$$

$\Rightarrow \frac{1}{2}(3)V_1{}^2 + (3)g(0) = \frac{1}{2}(3)(0)^2 + (3)(9.8)(10)$ (5)

$\Rightarrow V_1{}^2 = 196$

$\Rightarrow V_1 = 14\ m/s$ (5)

Speed of the bullet before impact

$$M_1U_1 + M_2U_2 = (M_1 + M_2)V$$

$\Rightarrow (0.1)U_1 + (2.9)(0) = (0.1 + 2.9)(14)$ (5)

$\Rightarrow U_1 = 420\ m/s$ (5)

(b)

Train A and Train B are on parallel tracks and travelling in opposite directions.
Train A starts from rest at Maynooth and accelerates uniformly at 0·5 m s^{-2} towards Leixlip to a speed of 25 m s^{-1}. It then continues at this constant speed.
At the same instant as train A is leaving Maynooth Train B passes through Leixlip heading towards Maynooth at a constant speed of 30 m s^{-1}.
Three minutes after leaving Leixlip train B starts to decelerate at 0·25 m s^{-2} and comes to rest at Maynooth.

(i) Find the distance between Maynooth and Leixlip.

(ii) At what distance from Maynooth do the trains meet?

After travelling at 25 m s^{-1} for a time, train A decelerates and comes to rest at Leixlip 36 seconds after train B stops at Maynooth.

(iii) Find the deceleration of train A.

(i) B $v^2 = u^2 + 2as$
$0 = 30^2 - 0.5s$
$s = 1800$

$s = 30 \times 180 + 1800$
$s = 7200$ m

$v = u + at$
$0 = 30 - 0.25t$
$t = 120$ (5)

$s = 30 \times 180 + 30 \times 120 + \frac{1}{2}\left(-\frac{1}{4}\right)120^2$
$s = 7200$ m (5)

(ii) A $v = u + at$
$25 = 0 + 0.5t$
$t = 50$ (5)

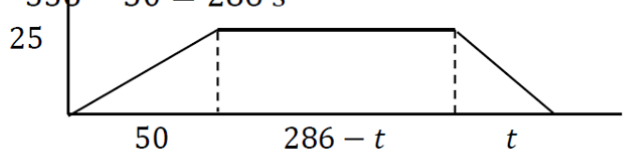
$s_1 = 0 \times 50 + \frac{1}{2}\left(\frac{1}{2}\right)50^2 + 25(t - 50)$
$s_1 = 25t - 625$

B $s_2 = 30t$

$s_1 + s_2 = 7200$ (5)
$25t - 625 + 30t = 7200$
$t = 142.27$

$s_1 = 25 \times 142.27 - 625$
$s_1 = 2931.75$ m (5)

(iii) B takes 300 s to stop
A takes 336 s to stop
$336 - 50 = 286$ s



A $7200 = \frac{1}{2} \times 50 \times 25 + (286 - t) \times 25 + \frac{1}{2} \times t \times 25$
$t = 46$
deceleration $= \frac{25}{46} = 0.54$ m s^{-2} (5) (30)