- Chapter 4: Newton's Laws and Connected Particles
- Topic 21: Newton's Laws of Motion

- Force/Mass:
  - You might recall learning about Forces in your Junior Cert Science course.
  - The definition you learned for a Force was:

    A Force is something that causes an object to speed up or slow down i.e. to accelerate.

  - The strength or magnitude of a Force was measured in newtons.
  - Forces also have a direction, so they are examples of vectors.
  - You might also remember learning about mass in Science, and the definition of mass was:

    Mass is the amount of matter in something.

  - The magnitude of the mass was measured in kilogrammes, or grammes.
  - As mass only has a magnitude, and no direction, it is a scalar quantity.
  - Another important point covered in Science was that mass and weight were not the same thing.
    - The weight of an object is a force experienced by the object, that pulls it towards the centre of the earth. This value could change depending on where the object was in the Solar System.
    - The mass of an object is the amount of matter, and remains the same, regardless of where the object is.
    - To calculate the weight of an object on the earth, you learned to multiply the mass by 10 (or more precisely now, $9.81 m/s^2$, which is the acceleration of an object towards the centre of the earth due to gravity)
- Momentum:
  - You probably have a good idea already what is meant by the momentum of an object, even though you wouldn't have come across it before.
  - If something is rolling down a hill, it "gathers momentum", and it is determined by both the mass and the velocity of the object.
  - In fact: **Momentum = Mass x Velocity**
  - The units we use for momentum are kg m/s.
  - We will come across momentum again later in the course.
- Tension:
  - Tension is a force exerted throughout strings and on objects by strings.
  - For example, if a light is hanging from the ceiling by a cable, the weight of the light is pulling it downwards, while the tension in the cable is applies an upwards force.

- Newton's Laws of Motion:
  - The scientist Isaac Newton established three very famous laws of motion in the 17^{th} century.
  - They are:

Law 1: A body will continue in a state of rest, or of uniform motion, in a straight line, unless it is impelled to change that state by an external force applied to it.
Law 2: The change in momentum per unit time is proportional to the applied force, and takes place along the straight line in which the force acts.
Law 3: To every action there is an equal and opposite reaction.

  - Newton's 2^{nd} Law can be simplified to the following rule:

F = ma

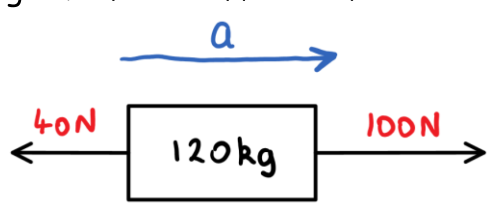
- Example 1: Pg 69 Ex 4A Q2

A boat has mass 120kg. A man pulls it in shallow water by means of a horizontal rope.
(i) The resistance to motion is 40 N. The tension in the rope is 100 N. Find the acceleration of the boat.
(ii) If the man's partner, who has a mass of 60 kg, gets into the boat, what will the acceleration become assuming the forces remain the same?

Solution:

i)

- Firstly, we will look at a diagram of the different forces acting on the car:



- We can see that the boat is moving to the right, so the resistance of 40N works against the direction of motion to the left
- So, the total force acting on the car is:

$F = 100 - 40 = 60$ N

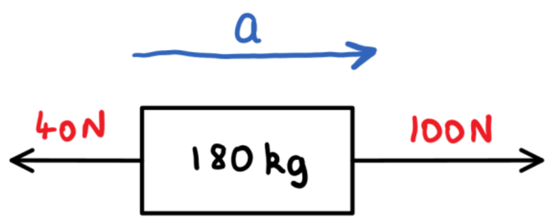
- We can now find the acceleration using Newton's 2^{nd} Law:

F = ma
=> 60 = 120a
=> a = $\frac{60}{120}$ = 0.5 m/s^2

- This means it will accelerate at 0.5 m/s^2 along the same line that it is being driven.

ii)

- Again, we will begin by drawing a diagram of the situation, showing all forces acting on the boat:



- The forces have remained the same so the overall force acting on the boat again is:

$F = 100 - 40 = 60$ N

- So, the acceleration using Newton's 2nd Law:

F = ma

=> 60 = 180a

=> a = $\frac{60}{180}$ = 0.33 m/s^2

- Example 2: Pg 69 Ex 4A Q8

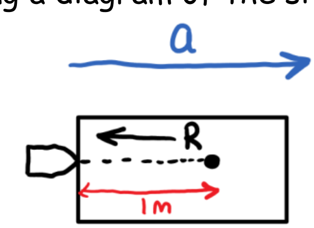
A bullet of mass 50 grams is travelling horizontally at 200 m/s. It enters wood and travels a distance of 1 m into the wood.

(i) Find the resistance of the wood, assuming it is uniform.

(ii) If the bullet had been travelling at twice that speed, how far into the wood would it have penetrated?

Solution:

- Again, we will begin by drawing a diagram of the situation:



i) In this part, we will first begin by calculating the acceleration of the bullet once it enters the block:

u = 200, v = 0, s = 1, a = ?

$v^2 = u^2 + 2as$

=> $(0)^2 = (200)^2 + 2a(1)$

=> $0 = 40{,}000 + 2a$

=> $2a = -40{,}000$

=> $a = -20{,}000$ m/s^2

- If we know the acceleration, then we can use Newton's 2nd law to calculate the overall force on the bullet:

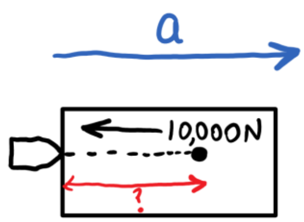
F = ma

=> F = (0.05)(-20,000)

=> F = -1000 N or 1 kN

- This means there is an overall force of 1000 N acting in the left direction on the bullet.
- From our diagram above, there is no other force acting on the bullet other than the resistance of the wood itself, so that resistance must be 1000 N or 1 kN.

ii) The diagram of this situation will be:



- As the resistance force supplied by the block hasn't changed, the deceleration of the bullet in the block will be the same as the first part, so:

u = 400 v = 0 a = -20,000, s = ??

$v^2 = u^2 + 2as$

=> $(0)^2 = (400)^2 + 2(-20{,}000)(s)$

=> $0 = 160{,}000 - 40{,}000s$

=> $40{,}000s = 160{,}000$

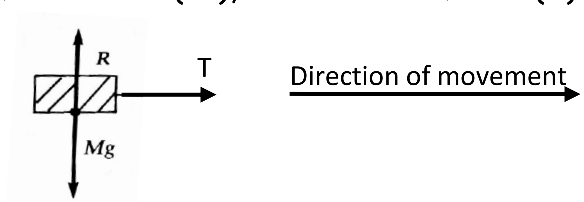
=> $s = 4m$

Classwork Questions: pg 69/70 Ex 4A Qs 1/3/4/7/9/10/12

➢ Topic 22: Normal Reaction and Friction

• Reaction Force:
- A reaction force is a force transmitted from one object to another by direct contact.
- For example, when you are standing on the ground, your weight is acting down into the ground, and the ground supports you by a normal reaction (R) upwards.
- If the ground is soft, then the reaction force isn't sufficient to support your weight, and hence you sink into the ground.

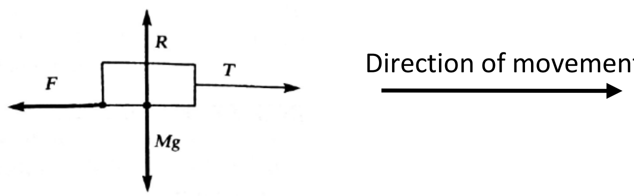
• Friction Force:
- If a block was being pulled along a **smooth** table, the forces acting on the block would be the weight of the block (W), the reaction force (R) and the tension in the string (T).



- If the table is **not** smooth however, a new force comes into play, called friction.
- You might remember learning a definition of friction from your Junior Science course:

> Friction is a force that opposes motion, between two bodies, in sliding contact with each other.

- The key words in the definition above are "opposes motion" so, friction always acts in the opposite direction to the direction of movement.
- So, if the block above was being pulled along a **rough** table, the forces acting on it would be:



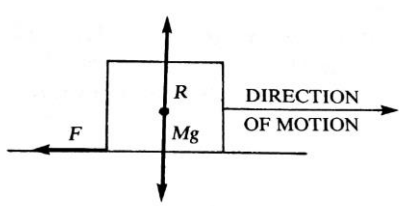
Important Note:

- The ratio $\frac{F}{R}$ is known as the coefficient of friction.
- We use the Greek letter μ ("mew") to denote it, so:

$$\mu = \frac{F}{R} \qquad \text{OR} \qquad F = \mu R$$

- The coefficient of friction depends on what the surface of the table is like, but it not affected by the **shape**, or **size** of the objects.

- Calculating Friction:
  - If we were dragging a box across a wooden floor, where the mass of the box was 100kg and the coefficient of friction between the box and the floor was 0.3, what is the amount of friction at play?
  - First, let's look at a diagram of all the forces:



  - As the piano doesn't move up or down, the forces acting upward must equal the forces acting downward:

=> R = Mg

=> R = (100)(9.8)

=> R = 980N

  - We know from our formula above:

$F = \mu R$

=> $F = (0.3)(980) = 294N$

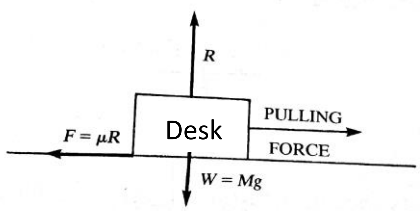
- An interesting thing to note here is that it will take a force of greater than 294N to move this box, but what happens if we apply a force smaller than 294N?
- Well, it turns out that the box will only call into play as much friction as it needs to use i.e. if we applied a force of 100N to the box, the friction force would only be 100N.
- The maximum amount of friction that can be called into play is known as the limiting friction, and in the example above, it would be 294N.
- Once we can apply a force **greater** than the limiting friction, we will be able to move the box.

• Example: Pg 73 Ex 4B Q2

A man pushes a desk of mass 50kg across a floor by applying a horizontal force of 300N. If the coefficient of friction between the desk and the floor is 0.6, find the acceleration of the desk.

Solution:

- As always, we will start by drawing a diagram showing all the forces at play:



- Since the desk doesn't move up or down, the force upwards must equal the force downwards:

=> R = Mg

=> R = (50)(9.8)

=> R = 490N

- The limiting friction so will be:

$F = \mu R$

=> $F = (0.6)(490) = 294N$

- So, the overall net force will be:

300 – 294 = 6N

- Now, we can use Newton's 2nd law to find the acceleration:

F = ma

=> 6 = (50)a

=> a = $\frac{6}{50} = \frac{3}{25}$ $or\ 0.12$ m/s^2

Show Q8 diagram and how to handle force at an angle, but leave rest until Particles on Slopes.

- We will now look at more complicated examples, involving inclined planes and pulling objects at angles, other than horizontal.

Classwork Questions: pg 73/74 Ex 4B Qs 3/4/5 (Can try Qs 8/10 as a challenge)

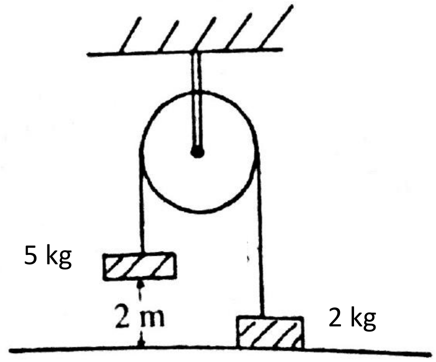
➢ Topic 23: Systems of Connected Particles

- By connected particles, it normally means 1 or more weights joined together by inelastic strings and there are pulleys involved also.

- **Example:** Pg 77 Ex 4C Q4

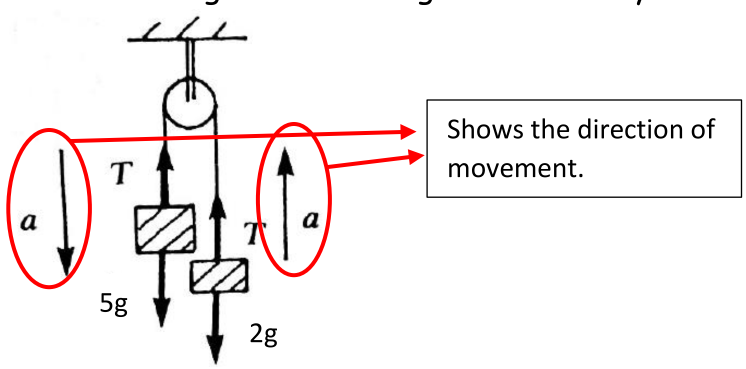
A fixed smooth pulley has masses of 2kg and 5kg hanging freely from either side by means of a light inextensible string. The string is released when the 2 kg mass is on a horizontal floor and the 5kg mass is 2 m above the floor. Find:

i) the common acceleration of the two masses
ii) their speed when the 5 kg mass hits the floor
iii) how much further will the 2kg mass then rise?



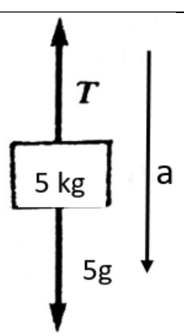
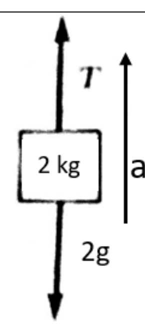
Solution:

- The diagram below shows the forces acting on the moving masses at any time:



i)

- We now handle the two particles separately:

5 kg mass		2 kg mass	
	As mass is moving upwards: 5g - T = ma (F = ma) 5g – T = 5a T + 5a = 5g….Eqn 1		As mass is moving downwards: T – 2g = ma (F = ma) T – 2g = 2a T - 2a = 2g…..Eqn 2

- We now solve the two equations together to find a:

$$\text{Eqn 1: } \quad T + 5a = 5g$$
$$\text{Eqn 2} \times -1: \quad -T + 2a = -2g$$
$$7a = 3g$$
$$\Rightarrow a = \frac{3g}{7} = \frac{3(9.8)}{7} = 4.2 \text{ m/s}^2.$$

ii)

- The 5 kg mass starts from rest, moves a distance of 2m with acceleration 4.2 m/s^2, so we can calculate its final speed using one of our equations of motion:

$u = 0, s = 2, a = 4.2\ v = ?$

$v^2 = u^2 + 2as$

$\Rightarrow v^2 = (0)^2 + 2(4.2)(2)$

$\Rightarrow v^2 = 16.8$

$\Rightarrow v = \sqrt{16.8} = 4.1$ m/s

iii)

- When the 5kg mass hits the ground, the string becomes loose and the 2kg mass is now a projectile, flying freely under gravity. Its initial speed is 4.1 m/s. When it reaches its highest point, its speed will be 0m/s. It has deceleration 9.8m/s^2 due to gravity slowing it down and we require the distance covered.

$u = 4.1, v = 0, a = -9.8, s = ?$

$v^2 = u^2 + 2as$

$\Rightarrow (0)^2 = (4.1)^2 + 2(-9.8)(s)$

$\Rightarrow 0 = 16.81 - 19.6s$

$\Rightarrow 19.6s = 16.81$

$\Rightarrow s = \frac{16.81}{19.6} = 0.86$ m.

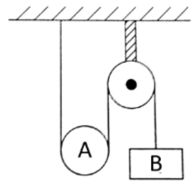
Day 1: Classwork Questions: pg 77/78 Ex 4C Qs 1/3/5 and then try Q6

Day 2: Classwork Questions: pg 77/78 Ex 4C Qs 7/8/10/11

- ## Topic 24: Movable Pulleys

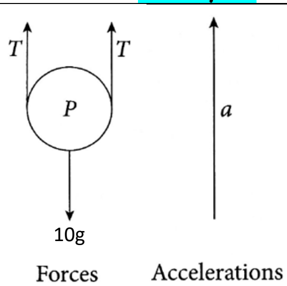
- **Example 1:** Pg 82 Ex 4D Q11

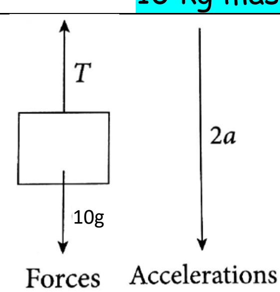
  A light inextensible string has one end fastened to a point on the ceiling. It then passes under a smooth movable pulley A of mass 10 kg, then over a smooth, light fixed pulley. A second particle B, of mass 10 kg, hangs freely from the other end of the string. All parts of the string which are not in contact with the pulleys are vertical. Find the acceleration of A in terms of g. Find also, the tension in the string in terms of g.



Solution:

- Our first problem, is to try and decide what way the system is going to move, when released from rest.
- To figure this out we carry out a small test:
  - Pulley A has two strings pulling it up, but mass B only has one
  - So, as the string mass is negligible (a "light" string), we need to see which is bigger: 10 ÷ 2 or 10 ÷ 1?
  - As 10 ÷ 1 is a bigger number, the mass B will move downwards, and hence, pulley A will move upwards.
- We can now start looking at each particle in turn:

<table>
<tr><th>Pulley A</th><th>10 kg mass</th></tr>
<tr><td>

Using F = ma:
$2T - 10g = 10a$
=> $T - 5a = 5g$.......Eqn 1</td>
<td>

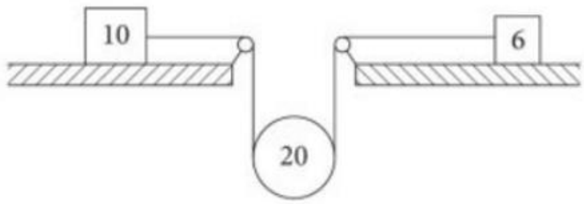
Using F = ma:
$10g - T = 10(2a)$
=> $-T - 20a = -10g$.......Eqn 2</td></tr>
<tr><td colspan="2">We now solve equations 1 and 2 together:
Eqn 1: $T - 5a = 5g$
Eqn 2: $-T - 20a = -10g$
$-25a = -5g$
=> $a = \frac{5g}{25} = \frac{g}{5}$ m/s² => T = 6g N (using Eqn 1 or 2)</td></tr>
</table>

Day 1: Classwork Questions: Pg 80 – 83 Qs 1 – 3/5/6/12/13/15

Demo of 2 particles and a movable pulley and relationship between accelerations.

- Example 2: Pg 83 Ex 4D Q18

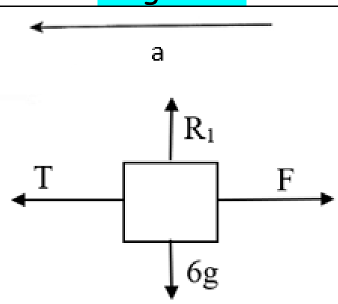
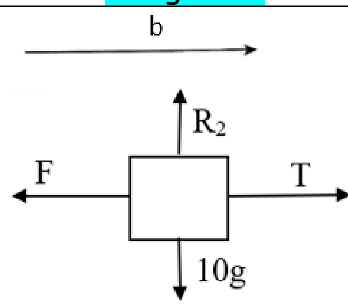
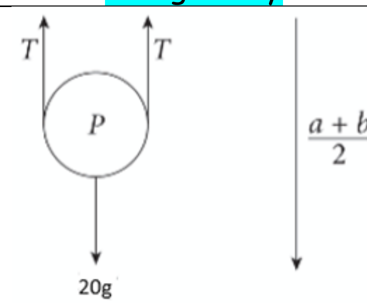
A particle of mass 10 kg rests on a horizontal table and another particle of mass 6 kg rests on another horizontal table, as shown. The particles are connected by a light inextensible string, which passes under a movable pulley of mass 20 kg. The coefficient of friction between each particle and its table is 0.5. Find the tension in the string and the acceleration of the particles and the pulley.



Demo of 2 particles and a movable pulley and relationship between accelerations.

Solution:

- Again, we can now start looking at each particle in turn:

6 kg Mass	10 kg mass	20 kg Pulley
		
Vertically F = ma $R_1 = 6g$ **Horizontally** F = ma $T - F = 6a$ => $T - \frac{1}{2}(6g) = 6a$ => $T - 3g = 6a$...I	**Vertically** F = ma $R_2 = 10g$ **Horizontally** F = ma $T - F = 10b$ => $T - \frac{1}{2}(10g) = 10b$ => $T - 5g = 10b$...II	Using F = ma: $20g - 2T = 20(\frac{a+b}{2})$ => $20g - 2T = 10a + 10b$ => $10g - T = 5a + 5b$ III

We now sub equations I and II into equation III:

Eqn III: $10g - T = 5a + 5b$

=> $10g - T = 5(\frac{T-3g}{6}) + 5(\frac{T-5g}{10})$

=> $300g - 30T = 25(T - 3g) + 15(T - 5g)$ (x 30)

=> $300g - 30T = 25T - 75g + 15T - 75g$

=> $300g + 75g + 75g = 30T + 25T + 15T$

=> $450g = 70T$

=> $T = \frac{450g}{70} = \frac{45g}{7}$ m/s² => T = 63 N

So, we can find a and b using equations I and II:

$a = \frac{T-3g}{6} = \frac{63-3(9.8)}{6} = 5.6$ m/s² and $b = \frac{T-5g}{10} = \frac{63-5(9.8)}{10} = 1.4$ m/s²

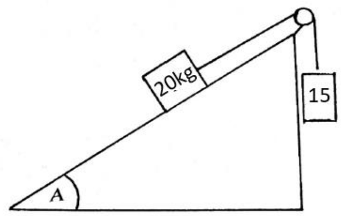
Which means the pulley has acceleration: $\frac{a+b}{2} = \frac{5.6+1.4}{2} = 3.5$ m/s²

Day 2: Classwork Questions: Pg 80 – 83 Qs 16/17/19/20

- Topic 25: Particles on Slopes

- Example: Pg 85 Ex 4E Q4

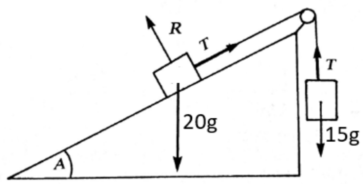
A particle of mass 20 kg is held on a smooth plane, which is inclined at an angle A to the horizontal, where $\tan A = \frac{4}{3}$. The particle is connected to another particle of mass 15 kg by means of a light inextensible string passing over a smooth pulley at the top of the plane. Find the common acceleration of the two particles, correct to 2 decimal places. Does the 15 kg mass rise or fall?



Solution:

- Let's look at all the forces acting on the two masses first:

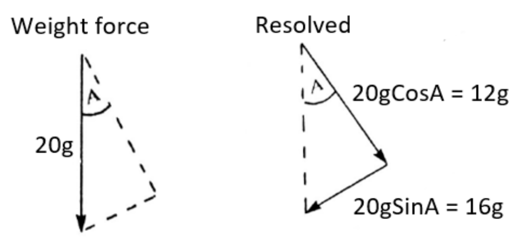
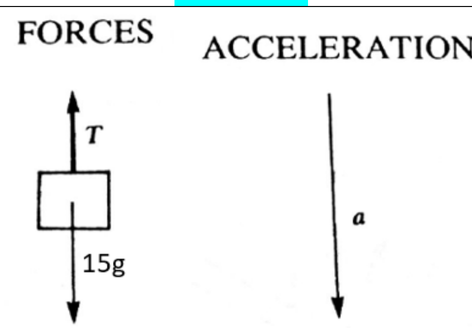
Let T = tension in the string and assume 15 kg mass falls



- If $\tan A = \frac{4}{3}$

=> $\sin A = \frac{4}{5}$ and $\cos A = \frac{3}{5}$

- Again, we handle each mass separately:

Mass M	Mass 2M
We have to resolve the weight of the mass first:  Forces perpendicular to the plane: $R = 20g \cos A$ => $R = 12g$.......Eqn 1 Forces along the plane: $T - 20g \sin A = 20a$ => $T - 16g = 20a$......Eqn 2	 Forces Up/Down: $15g - T = 15a$..........Eqn 3

- We now solve Equations 2 and 3 together:

Eqn 2: $T - 16g = 20a$

5 x Eqn 3: $-T + 15g = 15a$

=> $-g = 35a$

=> $a = \frac{-g}{35}$ m/s^2.

- As we got a negative value for 'a', it simply means we chose the wrong direction of movement when we started the question, so the 20kg mass actually slides down the plane, and the 15kg mass rises.

- Both particles accelerate at $\frac{g}{35}$ m/s^2 and the 15kg mass rises.

Classwork Questions: pg 85/86 Ex 4E Qs 2/3/5 and then try Q6

Revision Questions and Test.