Topic 5: Functions/Graphs

1) The Basics:

a) Terminology:

- **Domain** = the values that are put **into** a function.
- **Range** = the values that come **out** of a function.
- **Codomain** = the values that **could come out** of a function.

b) Notation:

The different ways functions are written are:

- $f(x) = x^2 + 3x$
- $f:x \rightarrow x^2 + 3x$
- $y = x^2 + 3x$

c) Evaluating Functions:

Example: If $f(x) = 2x^2 + 3$, find f(3) and f(-1).

$f(3) = 2(3)^2 + 3 = 21$

$f(-1) = 2(-1)^2 + 3 = 5$

d) Finding Inputs of Functions:

Example: If $f(x) = 5x - 3$, find the value of x for which $f(x) = 12$.

$f(x) = 12$

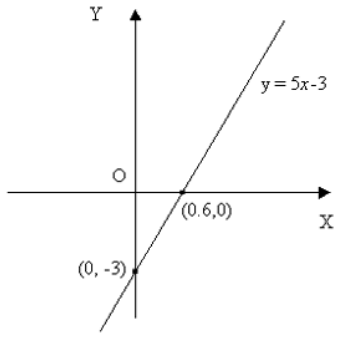
$\Rightarrow 5x - 3 = 12$

$\Rightarrow 5x = 15$

$\Rightarrow x = 3$

2) Types of Graphs:

a) Linear: $y = ax + b$

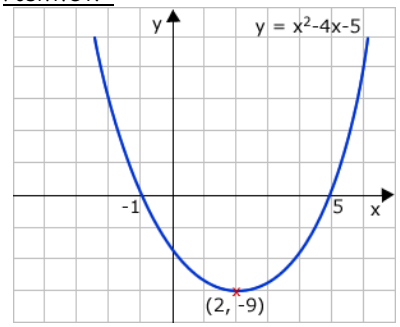


Notes:

- Graph above is for function of the form $y = ax + b$
- If 'a' is positive, the line increases from left to right but if 'a' is negative, the line decreases from left to right
- Function of the form $y = ax$ would be a line through the origin 'O'
- The root is where the graph crosses the x-axis….in the graph above, the root is 0.6.

b) Quadratic: $y = ax^2 + bx + c$

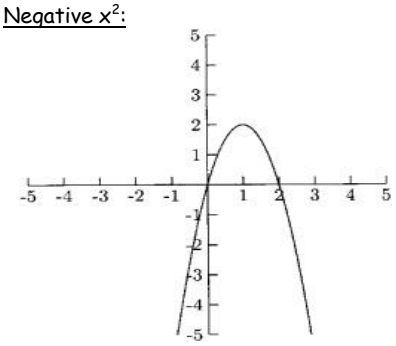
Positive x^2:



Notes:

- Graph above is for function of the form $y = ax^2 + bx + c$, where 'a' is a positive number
- Roots are where the graph crosses the x-axis…..in the graph above, the roots are -1 and 5
- The minimum point is the lowest point on the graph….in the graph above the minimum point is (2,-9).

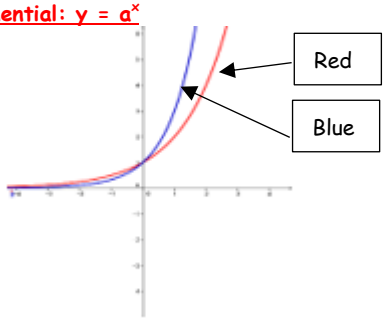
Negative x^2:



Notes:

- Graph above is for function of the form $y = ax^2 + bx + c$, where 'a' is a negative number
- Roots are where the graph crosses the x-axis…..in the graph above, the roots are 0 and 2
- The maximum point is the highest point on the graph…..in the graph above the maximum point is (1, 2)

c) Exponential: $y = a^x$



Notes:

- Red line in the graph above is graph of $y = 2^x$ and blue line is graph of $y = 3^x$
- All graphs of the form a^x pass through the point (0,1)
- Note that $y = 3^x$ rises at a steeper rate after passing through the point (0,1)
- Graphs of the form $a2^x$ would look similar to the graph of 2^x but would be shifted on the left

3) Drawing/Interpreting Graphs:

a) Drawing Graphs:

- Just fill in the values from the domain and use calculator.

Example: Draw the graph of $x^2 - 3x - 4$, in the domain $-2 \leq x \leq 1$

$f(x) = x^2 - 3x - 4$

$f(-2) = (-2)^2 - 3(-2) - 4 = 6$ $(-2, 6)$
$f(-1) = (-1)^2 - 3(-1) - 4 = 0$ $(-1, 0)$
$f(0) = (0)^2 - 3(0) - 4 = -4$ $(0, -4)$
$f(1) = (1)^2 - 3(1) - 4 = -6$ $(1, -6)$
$f(2) = (2)^2 - 3(2) - 4 = -6$ $(2, -6)$
$f(3) = (3)^2 - 3(3) - 4 = -4$ $(3, -4)$

Can plot these on graph paper. Should know shape of graph from Section 2.

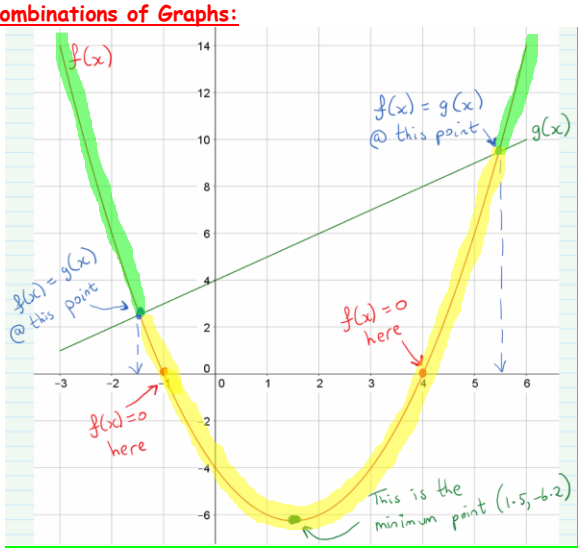
b) Interpreting Graphs:

Tip:

Use ruler and dotted lines when working out values from a graph

- To find f(2) or f(-1) from graph, for example: come up from x = 2 or x = -1 until you hit the graph and then go across to y value
- To find f(x) = 3 or f(x) = -2 from graph: draw a line through y = 3 or y = -2, and then come up/down to x-axis from the point(s) where the line crosses the graph
- Roots are where graph crosses x-axis i.e. f(x) = 0
- Axis of symmetry is the line that cuts the graph into 2. Only arises in U or ∩ shape.

c) Combinations of Graphs:



- f(x) = g(x) is where the two functions intersect
- Yellow Highlighted section is where f(x) < g(x)
- Green highlighted section is where f(x) > g(x)