End of Chapter Revision Sheet: Impacts & Collisions 5th Year

Level 1:

Q1. A particle of mass $0.2\ kg$ is fired horizontally with speed $200\ m/s$ into a block of sand that is at rest. The block is of mass $2\ kg$ and is sitting on a smooth horizontal surface. The particle emerges from the block with a speed of $120\ m/s$. Find the speed of the block.

Q2. A smooth sphere of mass $4\ kg$, moving with a speed of $11\ m/s$ collides directly with a smooth sphere of mass $6\ kg$, moving in the same direction with a speed of $7\ m/s$. After collision, the $6\ kg$ sphere moves with a speed of $10\ m/s$.
Calculate (i) the speed of the first sphere after collision and (ii) the coefficient of restitution.

Q3. Two elastic spheres of mass, $1\ kg$ and $2\ kg$, travelling in opposite directions collide directly. The speeds before collision are $16\ m/s$ and $9\ m/s$ respectively. If the coefficient of restitution is $\frac{5}{7}$, calculate the kinetic energy lost in the collision.

Q4. A smooth sphere A of mass M moving with speed $\sqrt{13}\ m/s$ collides with a smooth sphere B of mass $2M$ which is at rest. The direction of motion of A makes an angle of $\tan^{-1}(\frac{2}{3})$ with the line of centres at impact. The coefficient of restitution is 0.5. Calculate:
(i) the velocity of each sphere after the collision
(ii) the loss in kinetic energy due to the impact
(iii) the impulse imparted to each sphere during impact

Q5. A particle A of mass m moving with speed $5u$ on a smooth horizontal plane collides directly with another particle B of mass $6m$ moving with speed u in the same direction. After this collision B strikes a smooth vertical wall at right-angles to the direction of its motion. The coefficient of restitution between the two particles is $\frac{3}{4}$ and the coefficient of restitution between B and the wall is $\frac{1}{4}$.
(i) Find the velocities of A and B after the first collision.
(ii) Show that A and B to not collide again.

Q6. A ball falls from rest onto smooth horizontal ground from a height of $3.5\ m$. The ball hits the ground with a speed of $v\ m/s$ and rebounds back up to a height of s metres above the ground. If the coefficient of restitution between the ball and the ground is 0.6, find:
(i) the value of v (ii) the value of s.

Q7. A smooth sphere A, of mass $2\ kg$, is moving with a speed of $4\ m/s$. It collides directly with a smooth sphere B, of mass $5\ kg$, moving in the opposite direction with speed $2\ m/s$, on a smooth horizontal surface. Sphere B is brought to rest because of the collision and the coefficient of restitution for the collision is e.
(i) Find the speed of sphere A after the collision.
(ii) Find the value of e.
(iii) Find the percentage loss in kinetic energy due to the collision.

Level 2:

Q8. A train of mass $100\ tonnes$ is travelling up a straight track, which is at an angle of $5°$ to the horizontal. There is a force resisting the motion of the train of magnitude $6\ kN$ and the train's engine is working at a steady rate of $400\ kW$.
(i) Find the maximum speed of the train.
The track then becomes horizontal. The engine continues to work at the same rate of $400\ kW$ and the resistance to motion stays at $6kN$.
(ii) Find the acceleration of the train when it starts on the horizontal section.

Q9. A van of mass $2500\ kg$ accelerates from a stationary position on a straight horizontal road. There is a resistance to motion of $400\ N$ throughout, while the engine is working at a constant power of $21\ kW$.

(i) Find the acceleration of the car when its speed is $15\ m/s$.

(ii) Find the maximum speed the car can achieve at a power of $21\ kW$.

Q10. A smooth sphere of mass $1\ kg$ strikes a stationary sphere of mass $2\ kg$. The line of centres makes an angle of $30°$ with the original direction. If the collision is perfectly elastic, show that the spheres have equal speeds after impact.

Q11. A sphere of mass m and velocity $2u$ impinges directly on a sphere of mass $2m$ and velocity u, moving in the same direction. Prove that the velocity of the second sphere must have its value increased. Find e if the speed of mass m after the impact is reduced to u.

Q12. A smooth sphere A collides with an identical smooth sphere B which is rest. The velocity of A before impact makes an angle α with the line of centres at impact. Show that the angle θ through which the path of A is deflected is given by

$$\tan\theta = \frac{\tan\alpha\,(1+e)}{(1-e)+2\tan^2\alpha}$$

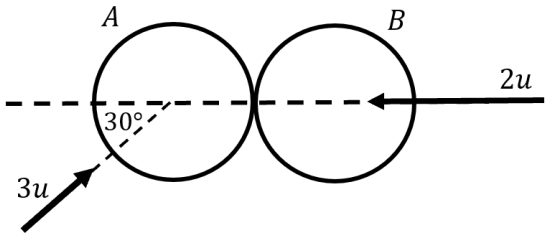
where e is the coefficient of restitution for the impact.

Q13. A smooth sphere P, of mass km, moving with speed u collides directly with a smooth sphere Q, of mass m, moving in the same direction with speed ku. Sphere P is brought to a stop by the collision.

(i) Find the speed of Q after the collision, in terms of k and u.

(ii) Prove that $k \leq \frac{1}{3}$.

Q14.

A smooth sphere A, of mass $3m$, collides obliquely with a smooth sphere B, of mass $2m$. The directions of motion and velocities of both spheres before the collision is shown in the diagram on the right. After the collision, A and B move in directions that are at right angles to one another. Find the coefficient of restitution for the collision.	

Q15. Two smooth spheres of mass $2m$ and m are moving in opposite directions with speed u and $2u$ respectively. They collide directly on a smooth horizontal table. If E and F are the sums of the kinetic energies of the spheres before and after the collision respectively, prove that $e = \sqrt{\frac{F}{E}}$.

Level 3:

Q16. A particle P of mass $2m$ is moving in a straight line with speed u at the instand when it collides directly with a particle Q of mass m which is at rest. The coefficient of restitution between P and Q is e.

(i) Show that after the collision P is moving with speed $\frac{1}{3}(2-e)u$.

(ii) Show that the kinetic energy lost in the collision is $\frac{1}{3}mu^2(1-e^2)$.

Q17. A smooth sphere P of mass M moving with a speed of v impinges on a smooth sphere Q of mass $2M$ which is at rest, the direction of motion of P making an angle of $30°$ with the line of centres. After impact P is moving at right angles to its original direction. Find the coefficient of restitution for the impact.

Q18. A particle of mass m is thrown vertically upwards with speed u from a point P on horizontal ground. Simultaneously a second identical particle, also of mass m, is thrown vertically downwards with speed from point Q, where Q is vertically above P at a distance of h, where $h < \frac{4u^2}{g}$. On impact the two particles adhere and move subsequently as a single particle. Show that the loss of kinetic energy due to the impact is mu^2. Show also that the speed with which the combined particle hits the ground is $\sqrt{gh}$.

Q19. A smooth sphere moves on a smooth horizontal surface and strikes an identical smooth sphere lying at rest on the table at a distance of $2\ m$ from a vertical wall. Prove that the next impact between the spheres will take place at a distance $\frac{4e^2}{1+e^2}$ metres from the wall, where e is the coefficient of restitution for all impacts involved.

Answers:

Q1. $8\ m/s$	Q2. (i) $6.5\ m/s$ (ii) $e = \frac{7}{8}$	Q3. $102\ J$	Q4. (i) $(0\vec{\imath} + 2\vec{\jmath})\ m/s, (1.5\vec{\imath} + 0\vec{\jmath})\ m/s$ (ii) $\frac{9M}{4}\ J$ (iii) $3M\ (Ns)$	
Q5. (i) $-u\vec{\imath}, 2u\vec{\imath}$	Q6. (i) $8.29\ m/s$ (ii) $1.26\ m$		Q7. (i) $1\ m/s$ (ii) $\frac{1}{6}$ (iii) 96%	
Q8. (i) $4.38\ m/s$ (ii) $0.85\ m/s^2$		Q9. (i) $0.4\ m/s^2$ (ii) $52.5\ m/s$	Q11. (ii) $e = \frac{1}{2}$	Q13. (i) $2uk\ m/s$
Q14. $e = 0.83$	Q17. $e = 1$			

Past Exam Questions:

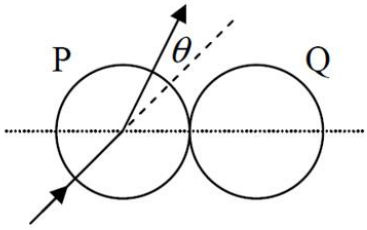
2012 Q5

(a) Three smooth spheres, A, B and C, of mass $3m$, $2m$ and m lie at rest on a smooth horizontal table with their centres in a straight line. Sphere A is projected towards B with speed 5 m s^{-1}. Sphere A collides directly with B and then B collides directly with C.

The coefficient of restitution between the spheres is e.

Show that if $e > \frac{3-\sqrt{5}}{2}$ there will be no further collisions.

(b) A smooth sphere P collides with an identical smooth sphere Q which is at rest. The velocity of P before impact makes an angle α with the line of centres at impact, where $0° \leq \alpha < 90°$.



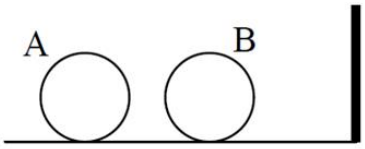
The velocity of P is deflected through an angle θ by the collision.

The coefficient of restitution between the spheres is $\frac{1}{3}$.

Show that $\tan\theta = \frac{2\tan\alpha}{1+3\tan^2\alpha}$.

2014 Q5

(a) A smooth sphere A, of mass $2m$, moving with speed u collides directly with a smooth sphere B, of mass $7m$, which is at rest. B then collides with a vertical wall which is perpendicular to the direction of motion of the spheres.

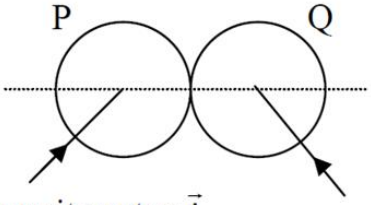


The coefficient of restitution is $\frac{1}{2}$ for all collisions.

(i) Show that the spheres will not collide for a second time.

(ii) What is the total loss of kinetic energy due to the impacts?

(b) A smooth sphere P, of mass $2m$, collides with a smooth sphere Q, of mass m. The velocity of P is $3u\,\vec{i} + 4u\,\vec{j}$ and the velocity of Q is $-4u\,\vec{i} + 3u\,\vec{j}$.



When they collide their line of centres is parallel to the unit vector $\vec{i}$.

The impact causes a loss of kinetic energy equal to $\frac{25mu^2}{2}$.

(i) Find the coefficient of restitution between the spheres.

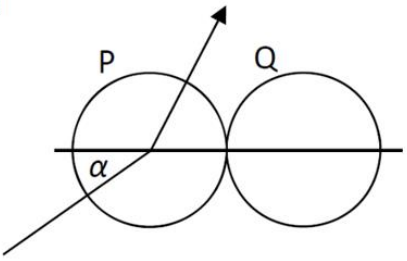
(ii) If the magnitude of the impulse imparted to each sphere due to the collision is kmu, find the value of k.

2017 Q5

(a) A small smooth sphere A, of mass 1·5 kg, moving with speed 6 m s^{-1}, collides directly with a small smooth sphere B, of mass m kg, which is at rest.
After the collision the spheres move in opposite directions with speeds v and $2v$, respectively.
80% of the kinetic energy lost by A as a result of the collision is transferred to B.
The coefficient of restitution between the spheres is e.

Find **(i)** the value of v
(ii) the value of e.

(b) A small smooth sphere P, of mass $3m$, collides obliquely with a small smooth sphere Q, of mass $7m$, which is at rest.



Before the collision the velocity of P makes an angle α with the line joining the centres of the spheres.
After the collision the speed of Q is v.
The coefficient of restitution between the spheres is $\frac{2}{7}$.

(i) Find, in terms of v and α, the **speed** of P before the collision.

(ii) If $\alpha = 30°$ find the angle through which the direction of motion of P is deflected as a result of the collision.

2019 Q5

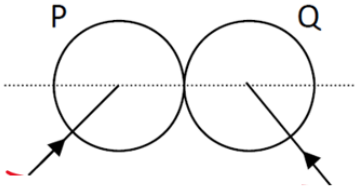
(a) A small smooth sphere A, of mass $3m$ moving with speed u, collides directly with a small smooth sphere B, of mass m moving with speed u in the opposite direction.
The coefficient of restitution between the spheres is $\frac{1}{2}$.

(i) Find, in terms of u, the speed of each sphere after the collision.

After the collision B hits a smooth vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{2}{5}$.
The first collision between the spheres occurred at a distance 2 metres from the wall.
The spheres collide again 4 seconds after the first collision between them.

(ii) Find the value of u.

(b) A smooth sphere P, of mass $2m$, collides with a smooth sphere Q, of mass m.
The velocity of P is $3u\,\vec{i} + 4u\,\vec{j}$ and the velocity of Q is $-4u\,\vec{i} + 3u\,\vec{j}$, where $\vec{i}$ is along the line of centres at impact.



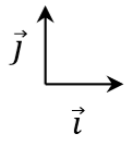
The coefficient of restitution between the spheres is $\frac{5}{7}$.

Find
(i) in terms of u, the speed of each sphere after the collision

(ii) the angle between the directions of P and Q after the collision.

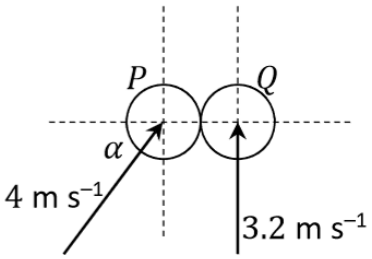
2023 Q2(b)

Two smooth spheres, P and Q, have equal radius and are of mass m and $2m$ respectively. P and Q collide obliquely. The line joining their centres at the point of impact lies along the $\vec{i}$ axis.



Before the collision, sphere P moves with a velocity of 4 m s^{-1} at an angle α with the $\vec{i}$ axis, where $\sin\alpha = \frac{4}{5}$.

Before the collision, sphere Q moves with a velocity of 3.2 m s^{-1} perpendicular to the $\vec{i}$ axis.



The coefficient of restitution between the spheres is e, where $0 \leq e \leq 1$.

Calculate, in terms of e, the velocity of each sphere immediately after they collide

Past Exam Questions:

2014: (a) (ii) $\frac{7}{8}mu^2$ (b) (i) 0.484 (ii) 6.93	2017: (a) (i) $\frac{12}{7}\ m/s$ (ii) $\frac{6}{7}$ (b) (i) $\frac{70v}{27\cos\alpha}$ (ii) $50.17°$
2019: (i) $\frac{u}{4}, \frac{5u}{4}$ (ii) 0.93 (b)(i) $u\sqrt{17}, 5u$ (ii) $67.17°$	2023: $v_P = 0.8(1-2e)\vec{i} + 3.2\vec{j}, v_Q = 0.8(1+e)\vec{i} + 3.2\vec{j}$