Question 6 **(30 marks)**

(a) $h(x) = x^2 + bx - 12$, where $x \in \mathbb{R}$ and b is a constant.

Find the value of b for which $x - 4$ is a factor of $h(x)$.

(b) Write the following expression as a single fraction in terms of t:

$$\frac{4}{2t+1} - \frac{7}{12t}$$

Question 1 **(30 marks)**

(a) Solve the following equation for $n \in \mathbb{N}$:

$$n - 3 = \sqrt{3n+1}$$

(c) Solve the following simultaneous equations for $x, y, w \in \mathbb{Z}$:

$$\begin{aligned} x + 2y &= 143 \\ y + 3w &= -74 \\ 4x + 5w &= 4 \end{aligned}$$

Question 1 **(30 marks)**

(a) Find the two values of $m \in \mathbb{R}$ for which $|5 + 3m| = 11$.

(c) $x^2 - px + 1$ is a factor of $x^3 - 2x - 3r$, where $p, r \in \mathbb{R}$ and $p < 0$.

Find the value of p and the value of r.

$p =$ _____ and $r =$ _____

(b) For the real numbers h, j, and k:

$$\frac{1}{h} = \frac{k}{j+k}$$

Express k in terms of h and j.

(b) $f(x) = 2x^3 - 21x^2 + 40x + 63$, where $x \in \mathbb{R}$.

(i) $x + 1$ is a factor of $f(x)$. Find the three values of x for which $f(x) = 0$.

(ii) The areas of the three regions **K**, **L**, and **N** give the following three equations (including the equation from **part (b)(i)**):

$$\begin{aligned} 4a + 3b + 3c &= 807 \\ 28a + 9b + 3c &= 879 \\ 76a + 15b + 3c &= 663 \end{aligned}$$

Solve these equations to find the values of a, b, and c.

Question 1 **(30 marks)**

(a) Find the two values of $m \in \mathbb{Z}$ for which the following equation in x has exactly **one** solution:

$$3x^2 - mx + 3 = 0$$

(b) Explain why the following equation in x has **no** real solutions:

$$(2x + 3)^2 + 7 = 0$$

(c) **(i)** Show that $x = -1$ is **not** a solution of $3x^2 + 2x + 5 = 0$.

(ii) Find the **remainder** when $3x^2 + 2x + 5$ is divided by $x + 1$.

That is, find the value of c when $3x^2 + 2x + 5$ is written in the form

$$3x^2 + 2x + 5 = (x + 1)(ax + b) + c$$

where $a, b, c \in \mathbb{Z}$.

Remainder, $c =$_____

Question 2 **(30 marks)**

(a) Given that $x = -3$ is a solution to $|x + p| = 5$, find the two values of p, where $p \in \mathbb{Z}$.

(b) $(x + 4)$ is a factor of $f(x) = x^3 + qx^2 - 22x + 56$, where $x \in \mathbb{R}$ and $q \in \mathbb{Z}$.

Show that $q = -5$, **and** find the three roots of $f(x)$.

Show:

Question 1 **(25 marks)**

(a) $f(x) = x^2 + 5x + p$ where $x \in \mathbb{R}$, $-3 \leq p \leq 8$, and $p \in \mathbb{Z}$.

(i) Find the value of p for which $x + 3$ is a factor of $f(x)$.

(ii) Find the value of p for which $f(x)$ has roots which differ by 3.

(b) Find the range of values of x for which $|2x + 5| - 1 \leq 0$, where $x \in \mathbb{R}$.

Question 3 **(25 marks)**

(a) Factorise fully: $3xy - 9x + 4y - 12$.

(b) Solve the equation $\frac{3}{2x+1} + \frac{2}{5} = \frac{2}{3x-1}$ where $x \neq -\frac{1}{2}, \frac{1}{3}$, and $x \in \mathbb{R}$.

(b) Solve the inequality $\frac{2x-3}{x+2} \geq 3$, where $x \in \mathbb{R}$ and $x \neq -2$.

Question 1 **(25 marks)**

(a) Solve the simultaneous equations.

$$\begin{aligned} 2x + 3y - z &= -4 \\ 3x + 2y + 2z &= 14 \\ x - 3z &= -13 \end{aligned}$$

Question 2 **(25 marks)**

(a) Find the range of values of x for which $|x - 4| \geq 2$, where $x \in \mathbb{R}$.

(b) Solve the simultaneous equations:

$$x^2 + xy + 2y^2 = 4$$
$$2x + 3y = -1.$$

Question 2 **(25 marks)**

(a) Find the set of all real values of x for which $2x^2 + x - 15 \geq 0$.

Question 5 **(25 marks)**

(a) Solve the equation $x = \sqrt{x+6},\ x \in \mathbb{R}$.

Question 2 **(25 marks)**

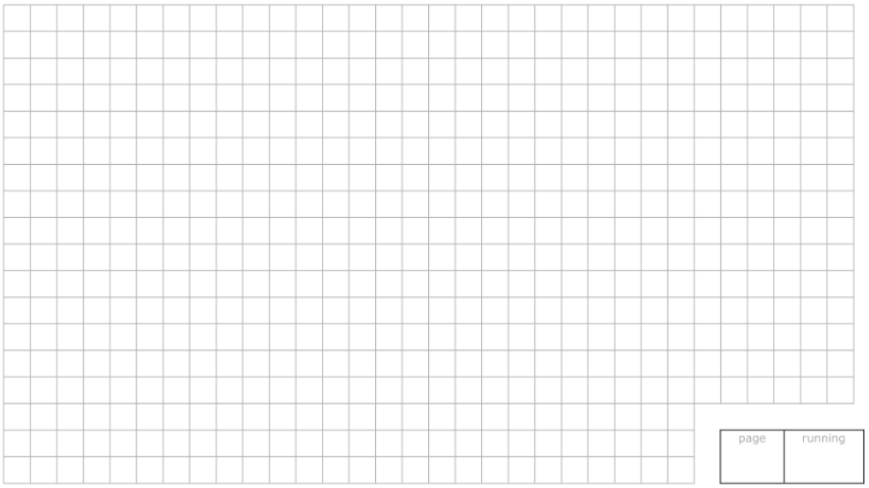
Solve the equation $x^3 - 3x^2 - 9x + 11 = 0$.

Write any irrational solution in the form $a + b\sqrt{c}$, where $a, b, c \in \mathbb{Z}$.

(b) Solve the simultaneous equations;

$$x+y+z=16$$
$$\frac{5}{2}x+y+10z=40$$
$$2x+\frac{1}{2}y+4z=21.$$

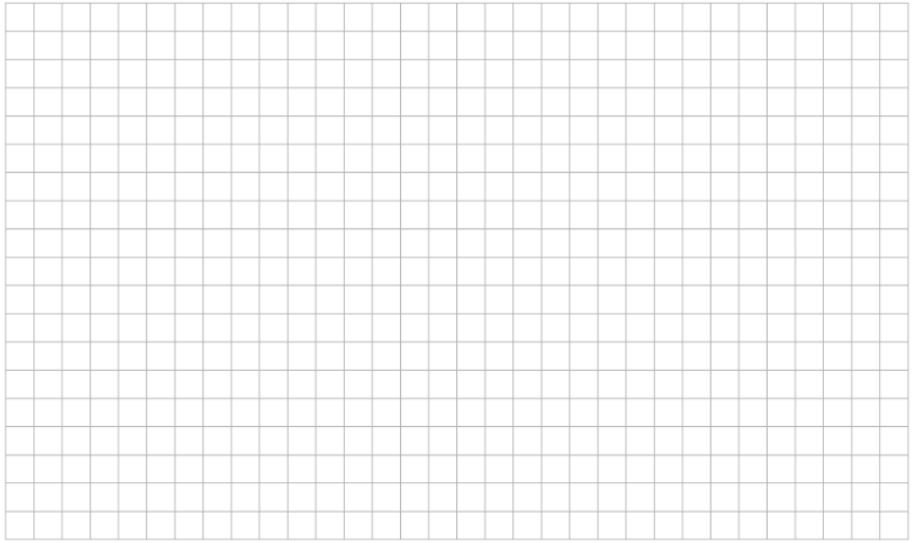
(b) Find the set of all real values of x for which $\frac{2x-5}{x-3}\leq\frac{5}{2}$.



Question 1 **(25 marks)**

(a) Solve the simultaneous equations:

$$a^2-ab+b^2=3$$
$$a+2b+1=0$$



(c) Solve the equation $x^2-2\sqrt{3}x-9=0$, giving your answers in the form $a\sqrt{3}$, where $a\in\mathbb{Q}$.