Topic 1: Arithmetic

1) The Basics:

a) Converting Units:

Steps:

1. Write the conversion with the unit you want on the right.
2. Get a 1 on the left-hand side, by dividing both sides.
3. Multiply both sides to get the value you want.

Example: If 1 inch = 2.54 cm, how many inches in 40cm?

Step 1: Put inches on the right

2.54cm = 1 inch

Step 2: Get a 1 on the left-hand side

1cm = $\frac{1}{2.54}$ inches (dividing both sides by 2.54)

Step 3: Multiply both sides

40cm = $\frac{1}{2.54} \times 40$ = 15.75 inches

b) Types of Numbers:

- **Natural (N)**: Positive Whole Numbers: e.g. 1, 2, 3, ...
- **Integers (Z)**: Positive and Negative Whole Numbers: e.g. -3, -2, -1, 0, 1, 2, 3,.....
- **Real (R):** All numbers: e.g.s -3, -1.4, 0.2, 6, 7/2, $\sqrt{8}$ .....
- **Rational (Q):** Numbers that **can** be written in the form $\frac{a}{b}$ e.g.s -5, 3, 1/2, -9/4.....
- **Irrational:** Numbers that **cannot** be written in the form $\frac{a}{b}$ e.g.s $\sqrt{3}$, $\sqrt{2}$, π .....
- **Prime:** A natural number bigger than 1 with only itself and 1 as divisors. e.g.s 2, 3, 5, 7, 11, 13, 17.....
- **Composite:** A number that is not prime. e.g.s., 6, 9, 15, 20

c) Rounding:

Rounding to Decimal Places:

- To round to 2 decimal places, we look at the 3rd number after the decimal point.
  - If it's 5 or more we round UP the 2nd number
  - If it's 4 or less we round DOWN the 2nd number
- Similar approach for rounding to other decimal places

Examples: i) 4.768 = 4.77 ii) 3.2745 = 3.27

Rounding to Significant Figures:

- To round to 3 significant figures, we look at the 4th significant figure.
  - If it's 5 or more we round UP the 3rd number and replace subsequent numbers with 0s
  - If it's 4 or less we round DOWN the 3rd number and replace subsequent numbers with 0s

Examples: i) 132,421 = 132,000 ii) 0.00472543 = 0.00473

d) Scientific Notation

Notes:

- A number is in scientific notation if it's in the form **$a \times 10^n$**, where 'a' has to be between 1 and 10.
  Examples: i) 3400 = 3.4×10^3 ii) 0.004 = 4×10^{-3}
- On a Casio calculator the button you will need to type in numbers in scientific notation is:

- To type in 7×10^4, press "7" and the button above and then "4"
- To convert numbers into scientific notation on your calculator:
  - Type in the number and press = to enter it on the screen.
  - Press "Shift" + "Mode" and select "Sci" from the menu. Then press "0".

e) HCF/LCM using Prime Factors:

Notes:

- When asked to find the HCF and LCM of 2 numbers using **prime factors**, use your calculator to find the prime factorisation of both numbers.
- Type in the number first and then press Shift + Button below.

- For **HCF:** pick the **lowest** power of each prime number.
- For **LCM:** pick the **highest** power of each prime number.

Example: Find the HCF and LCM of 60 and 80.

Using calculator:

$60 = 5^1 \times 3^1 \times 2^2$ $80 = 2^4 \times 5^1$

HCF:

- Lowest Power of 2 is 2^2
- Lowest Power of 3 is 3^1.....but 3^1 doesn't go into 80 so we exclude this
- Lowest Power of 5 is 5^1 => HCF of 60 and 80 is $2^2 \times 5^1 = 20$

LCM:

- Highest Power of 2 is 2^4
- Highest Power of 3 is 3^1

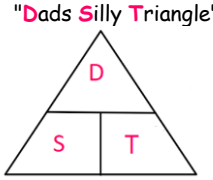
Highest Power of 5 is 5^1 => LCM of 60 and 80 is $2^4 \times 3^1 \times 5^1 = 240$

f) Speed, Distance and Time:

Notes:

- For all speed, distance and time calculations remember:

"Dads Silly Triangle"



If you want **Distance**, you cover the D in the triangle, so:

D = S x T (Units are usually m or km)

- If you want **Time**, you cover the T in the triangle, so:

T = D / S (Units are usually secs or hrs)

- If you want **Speed**, you cover the S in the triangle, so:

S = D / T (Units might be m/s or km/h)

- Average speed can be calculated using:

$$\text{Ave Speed} = \frac{Total\ Distance}{Total\ Time}$$

- Take care with units of time also. Remember 1hr 45mins = 1.75hrs and 90mins = 1.5hours.

g) Errors:

- The error is the difference between the estimated/measured value and the actual value.

$$\%\ \text{Error} = \frac{Error}{Actual\ Value} \times 100\ \%$$

h) Tolerances:

- The greatest variation in a measurement that can be allowed. To calculate it add/subtract one half of the precision of the measuring instrument to both sides.

Example: Ruler with precision of 0.3cm used to measure a table of length 200cm.

=> Tolerance interval = $200 \pm \frac{0.3}{2}$ = between 199.85 and 200.15

i) Ratio

Notes:

- Ratio shows how to break up a quantity proportionally.
- When given a ratio, add the values in the ratio together to get the total number of parts the quantity is being broken into.
- Write down the fraction each person gets.

Example: Divide €200 between Alan and Brian in the ratio 3:2.

3:2 means there are 3 + 2 = 5 parts

=> Alan gets $\frac{3}{5}$ and Brian gets $\frac{2}{5}$

=> Alan gets $\frac{3}{5}$ of €200 = €120

and Brian gets $\frac{2}{5}$ of €200 = €80

j) Foreign Exchange

Steps:

1. Write the conversion with the currency you want on the right.
2. Get a 1 on the left-hand side, by dividing both sides.
3. Multiply both sides to get the value you want.

Example: If €1 = $1.32, how many euro would you get for $200?

Step 1: Put euro on the right

$1.32 = €1

Step 2: Get a 1 on the left-hand side

$1 = € $\frac{1}{1.32}$ (dividing both sides by 1.32)

Step 3: Multiply both sides

$200 = $\frac{1}{1.32}$ x 200 = €151.52

2) Percentages/Profit/Loss/VAT:

a) Percentages:

To find the percentage of a number:

Example: Find 24% of 250.

Method 1: Calculate $\frac{24}{100} x \frac{250}{1} = 60$

Method 2: Multiply 250 by 0.24 = 60

To find the total when given percentage:

Example: 25% of the marks in an exam are going for the practical part. If there are 50 marks for the practical, how many marks is the whole exam worth?

Steps:

1) Let % = value you're given

25% = 50

2) Find what 1% represents by dividing both sides

1% = $\frac{50}{25} = 2$

3) Find 100% by multiplying by 100:

100% = 2 x 100 = 200marks

Note:

In this particular example, we could also have just multiplied 50 by 4, as 25% represents $^1/_4$ of the total marks

b) % Profit / Loss / Discount / Increase:

$$\% \text{ Profit or Mark-Up} = \frac{Profit}{Cost\ Price}\ x\ 100\ \%$$

$$\% \text{ Profit Margin} = \frac{Profit}{Selling\ Price}\ x\ 100\ \%$$

$$\% \text{ Loss} = \frac{Loss}{Cost\ Price}\ x\ 100\ \%$$

$$\% \text{ Discount} = \frac{Discount}{Cost\ Price}\ x\ 100\ \%$$

$$\% \text{ Increase} = \frac{Increase}{Original}\ x\ 100\ \%$$

c) VAT:

VAT excluded:

Example: Bill comes to €120. Find final bill with 13.5% VAT.

VAT = 13.5% of 120

= 120 x 0.0135 = €16.20

=> Final Bill = €120 + €16.20 = €136.20

VAT included:

Example: Bill including VAT comes to €340.50. Find bill without VAT, if VAT is 13.5%.

Bill + VAT = €340.50

=> 113.5% = €340.50

=> 1% = €3

=> 100% = €300

3) Income Tax:

a) Income Tax Terminology:

- **Gross Income:** total pay someone gets before any taxes or deductions are taken
- **Net Income:** Take home pay or pay that we get after all taxes and deductions
- Rates Of Tax: **Higher** Rate (usually about 42%) and **Standard** Rate (usually about 20%)
- **Standard Rate Cut-Off Point:** Anything you earn up to this is taxed at the standard rate of tax
- **Gross Tax:** Total tax owing to the government before credits are deducted
- **Tax Credits:** Money deducted from the gross tax
- **Tax Payable:** Tax that you actually pay after credits have been subtracted
- **Statutory Deductions:** Payments that you have to make to the government e.g. income tax (P.A.Y.E.)
- **Non-statutory Deductions:** Voluntary deductions that somebody pays e.g.s trade union fees or health insurance

b) Answering Questions:

- The questions are nearly always made up of 3 parts:
  - **Part 1:** Calculation of Gross Tax by adding….
    Tax @ Lower Rate + Tax @ Higher Rate
  - **Part 2:** Calculation of Tax Payable using the equation
    Tax Paid = Gross Tax - Tax Credits
  - **Part 3:** Working out Net Income by taking off all deductions including Tax Paid, USC and PRSI (See below), Union Fees, Health Insurance etc.

c) USC/PRSI:

USC: Have to be given the rates and bands. Sample calc below:

- 2% of the first €10036 = €200.72
- 4% of the next €5980 = €239.20
- 7% on the balance of income => need to subtract (10036 + 5980) from Gross Income and then get 7% of that figure

PRSI:

- Usually in class A, €127/week is free of PRSI deductions
  => €127 x 52 = $6604 (needs to be taken from gross income)
- Then pay 4% on the remainder of your income.

4) Compound Interest/Depreciation:

a) Terminology:

- **Principal:** Amount of money invested or borrowed
- **Interest:** Money added by the bank
- **Rate:** what percentage the interest is added at
- **Amount or Final Value:** The value of money at the end of the term it has been borrowed or invested for.

c) Depreciation: (items losing value)

- The formula on the right can be used for Depreciation problems…….just replace the '+' with a '-'……see below.
- The rate must be the same each year to use the formula.

$$F = P(1 - i)^t$$

where **F** is the Final value, **P** is the starting value, **i** is the Rate of Depreciation as a **decimal** (e.g. 2.5% = 0.025) and **t** is the time in years.

b) Answering Compound Interest Questions:

Method 1: Used if rates change from year to year or payments/withdrawals are being made between years

- Lay out Year 1, Year 2, Year 3 etc.
- Work out interest each year and add to Principal at start of the year

Method 2: Formula

See Tables pg 30

$$F = P(1 + i)^t$$

where **F** is the Amount, **P** is the Principal, **i** is the Rate of Interest as a **decimal** (e.g. 3% = 0.03) and **t** is the time in years the money is invested/borrowed for.

5) Household Bills:

Notes:

- With utility bills (e.g.s. gas, electricity, water) there is usually a unit rate i.e. a charge per unit used
- To calculate the units used, subtract the previous units reading from the current units reading
- With many bills there is also a standing charge that has to be added on.
- VAT is also added to the bills.
- With Gas Bills, there is also a Carbon Tax that needs to be added on.

Example: Calculate the cost of electricity if the previous meter reading was 21310 and the current reading is 21836, with a standing charge of €21.60. The cost per unit is €0.15 and VAT of 13.5% is added on.

Units used = Current Reading - Previous Reading
= 21836 – 21310 = 526 units
Cost for electricity = 526 x €0.15 = €78.90
Standing Charge = €21.60
=> Total Before VAT = €78.90 + €21.60 = €100.50
VAT = 13.5% of €100.50 = €13.57
=> Final Bill = €100.50 + €13.57 = €114.07