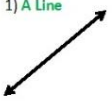

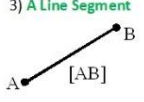






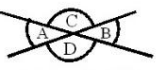

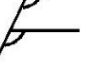

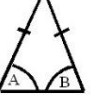
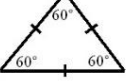

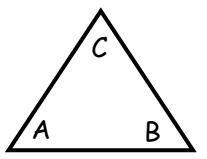
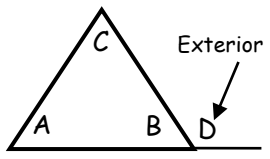
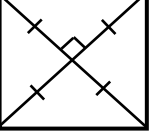
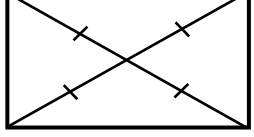
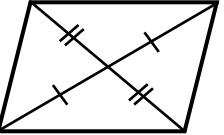
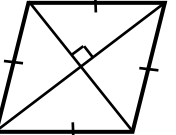
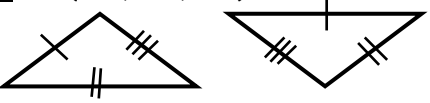
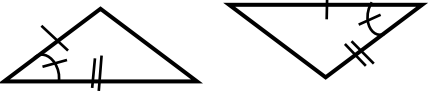
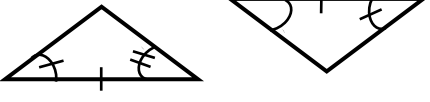
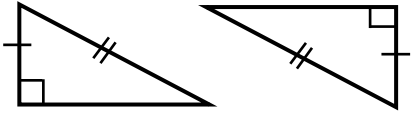
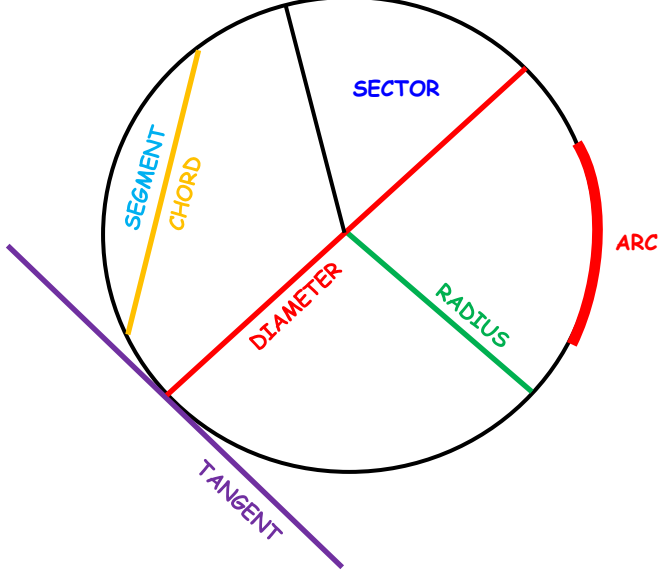
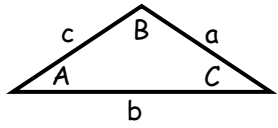
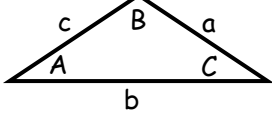
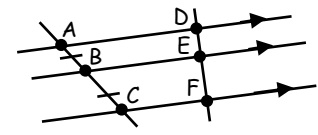
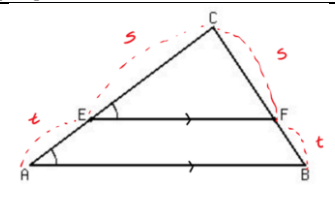
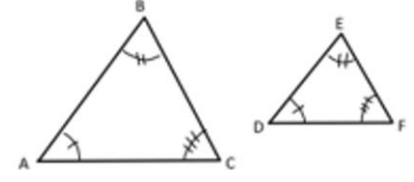
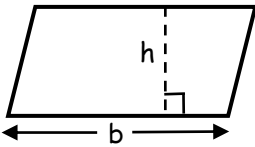
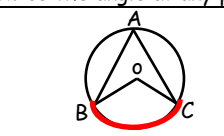
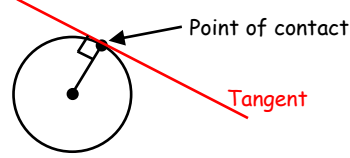
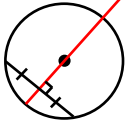


# Topic 9: Geometry

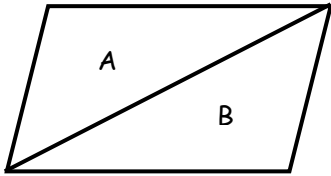
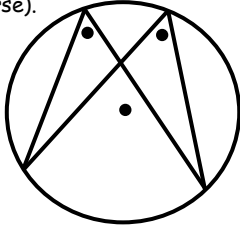
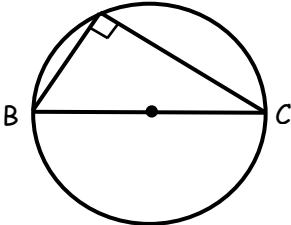
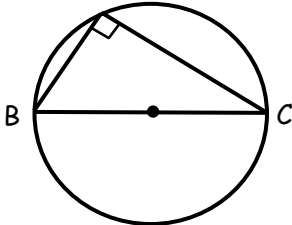
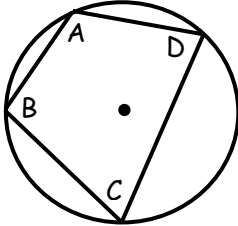
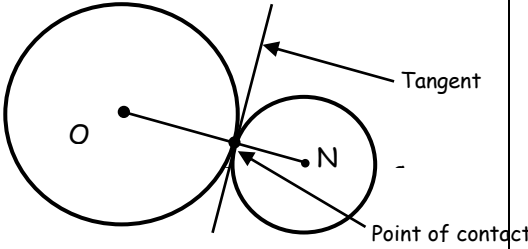
## 1) The Basics:

<p><b>a) Terminology:</b></p> <p><u>Lines:</u></p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>1) A Line</p>  </div> <div style="text-align: center;"> <p>2) A Half-line</p>  </div> <div style="text-align: center;"> <p>3) A Line Segment</p>  </div> </div> <p><u>Angles:</u></p> <div style="display: grid; grid-template-columns: repeat(4, 1fr); gap: 10px;"> <div style="text-align: center;"> <p>1) Acute Angle [angle between 0° and 90°]</p>  </div> <div style="text-align: center;"> <p>2) Obtuse Angle [angle between 90° and 180°]</p>  </div> <div style="text-align: center;"> <p>3) Reflex Angle [angle between 180° and 360°]</p>  </div> <div style="text-align: center;"> <p>4) Right Angle [Angle of 90°]</p>  </div> </div> <div style="display: grid; grid-template-columns: repeat(3, 1fr); gap: 10px; margin-top: 10px;"> <div style="text-align: center;"> <p>5) Straight Angle [Angle of 180°]</p>  </div> <div style="text-align: center;"> <p>6) Full Angle [Angle of 360°]</p>  </div> <div style="text-align: center;"> <p>7) Vertically Opposite Angles [‘X’ Shape: A = B and C = D]</p>  </div> </div> <div style="display: grid; grid-template-columns: repeat(3, 1fr); gap: 10px; margin-top: 10px;"> <div style="text-align: center;"> <p>8) Alternate Angles [‘Z’ Shape: A = B]</p>  </div> <div style="text-align: center;"> <p>9) Corresponding Angles [‘F’ Shape: A = B]</p>  </div> <div style="text-align: center;"> <p>10) Interior Angles [‘C’ Shape: A + B = 180°]</p>  </div> </div> <p><u>Triangles:</u></p> <div style="display: grid; grid-template-columns: repeat(3, 1fr); gap: 10px;"> <div style="text-align: center;"> <p>1) Isosceles [2 Equal Sides &amp; A = B]</p>  </div> <div style="text-align: center;"> <p>2) Equilateral [3 Equal Sides &amp; 3 Equal Angles of 60°]</p>  </div> <div style="text-align: center;"> <p>3) Scalene [No equal sides or angles]</p>  </div> </div>	<p><b>b) Properties of Triangles:</b></p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  <p><math>A + B + C = 180^\circ</math></p> </div> <div style="text-align: center;">  <p>Exterior Angle <math>D = A + C</math></p> </div> </div>
<p><b>c) Properties of Quadrilaterals:</b></p>	
<div style="text-align: center; margin-bottom: 10px;">  <p><b>Square</b></p> <ul style="list-style-type: none"> <li>- 4 equal sides</li> <li>- 4 equal angles of 90°</li> <li>- Opp sides are parallel</li> <li>- Diagonals bisect at 90° angles</li> </ul> </div> <div style="text-align: center; margin-bottom: 10px;">  <p><b>Rectangle</b></p> <ul style="list-style-type: none"> <li>- Opp sides are equal &amp; parallel</li> <li>- 4 equal angles of 90°</li> <li>- Diagonals bisect each other</li> </ul> </div> <div style="text-align: center; margin-bottom: 10px;">  <p><b>Parallelogram</b></p> <ul style="list-style-type: none"> <li>- Opp sides are equal &amp; parallel</li> <li>- Opp angles are equal</li> <li>- Diagonals bisect each other</li> </ul> </div> <div style="text-align: center;">  <p><b>Rhombus</b></p> <ul style="list-style-type: none"> <li>- 4 equal sides &amp; opp sides are parallel</li> <li>- Opp angles are equal</li> <li>- Diagonals bisect at 90° angles</li> </ul> </div>	
<p><b>d) Congruent Triangles</b></p> <ul style="list-style-type: none"> <li>• Triangles that sit exactly on top of each other</li> <li>• Matching sides are called <b>corresponding sides</b></li> </ul> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p><b>Type 1: SSS (Side, Side, Side)</b></p>  </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p><b>Type 2: SAS (Side 1, Angle in between sides 1 and 2, Side 2)</b></p>  </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p><b>Type 3: ASA (Angle 1, Side in between angles 1 and 2, Angle 2)</b></p>  </div> <div style="border: 1px solid black; padding: 5px;"> <p><b>Type 4: RHS (Right Angle, Hypotenuse, One other side)</b></p>  </div>	<p><b>e) Circle Terminology:</b></p> 

**2) Theorems:**

<p><b>a) Theorem:</b></p> <ol style="list-style-type: none"> <li>Vertically opposite angles are equal in measure.</li> <li>In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.</li> <li>If a transversal makes equal alternate angles on two lines then the lines are parallel, (and converse).</li> </ol>	<ol style="list-style-type: none"> <li>The angles in any triangle add to <math>180^\circ</math>.</li> <li>Two lines are parallel if and only if, for any transversal, the corresponding angles are equal.</li> <li>Each exterior angle of a triangle is equal to the sum of the interior opposite angles.</li> </ol>
<p>7. In a triangle, the largest angle is opposite the largest side, and the smallest angle is opposite the smallest side.</p> <div style="display: flex; align-items: center;">  <div style="border: 1px solid black; border-radius: 50%; padding: 10px; margin-left: 20px; background-color: #fff9c4;"> <p>If a is the largest side, then A is the largest angle, and vice versa.</p> </div> </div>	<p>8. In a triangle, two sides of a triangle are always greater than the third.</p> <div style="display: flex; align-items: center;">  <div style="border: 1px solid black; border-radius: 50%; padding: 10px; margin-left: 20px; background-color: #fff9c4;"> <p> <math>a + b &gt; c</math>  <math>b + c &gt; a</math>  <math>a + c &gt; b</math> </p> </div> </div>
<p>9. In a parallelogram, opposite sides are equal and opposite angles are equal (and converses).</p>	<p>10. The diagonals of a parallelogram bisect each other.</p>
<p>11. If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.</p> <div style="display: flex; align-items: center;">  <div style="border: 1px solid black; border-radius: 50%; padding: 10px; margin-left: 20px; background-color: #fff9c4;"> <p>                     If <math> AB  =  BC </math>  <math>\Rightarrow  DE  =  EF </math> </p> </div> </div>	<p>12. Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio s:t, then it also cuts [AC] in the same ratio.</p> <div style="display: flex; align-items: center;">  <div style="border: 1px solid black; border-radius: 50%; padding: 10px; margin-left: 20px; background-color: #fff9c4;"> <p>                     If EF is parallel to AB  <math>\Rightarrow \frac{ AE }{ CE } = \frac{ BF }{ CF }</math>                      or <math>\frac{ AE }{ AC } = \frac{ BF }{ BC }</math> </p> </div> </div>
<p>13. If two triangles are similar, then their sides are proportional, in order (and converse).</p> <div style="display: flex; align-items: center;">  <div style="margin-left: 20px;"> <p> <math>\frac{ AB }{ DE } = \frac{ BC }{ EF } = \frac{ AC }{ DF }</math>                      OR  <math>\frac{ DE }{ AB } = \frac{ EF }{ BC } = \frac{ DF }{ AC }</math> </p> </div> </div>	<p>14. [Theorem of Pythagoras] In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.</p>
<p>15. If the square of one side of a triangle is the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.</p>	<p>16. For a triangle, base x height doesn't depend on the choice of base.</p> <p>17. A diagonal of a parallelogram bisects the area.</p>
<p>18. The area of a parallelogram is base x height.</p> 	<p>19. The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on that arc.</p> <div style="display: flex; align-items: center;">  <div style="border: 1px solid black; border-radius: 50%; padding: 10px; margin-left: 20px; background-color: #fff9c4;"> <p><math> \angle BOC  = 2( \angle BAC )</math></p> </div> </div>
<p>20. Each tangent to a circle is perpendicular to the radius at the point of contact.</p> 	<p>21. The perpendicular from the centre of a circle to a chord bisects the chord.</p> 
<p><b>b) Other Theorem Terminology:</b></p> <ol style="list-style-type: none"> <li><b>Collinear</b> points are points that lie on the same line.</li> <li>An <b>axiom</b> is a statement that we accept without any proof: e.g. There is exactly one line through any two given points.</li> <li>A <b>theorem</b> is a rule that you can prove by following a certain number of logical steps or using a previous theorem or axiom. E.g. Pythagoras' Theorem</li> <li>A <b>proof</b> is a series of logical steps that we use to show a theorem is true.</li> </ol>	<ol style="list-style-type: none"> <li>A <b>corollary</b> is a statement that follows readily from a previous theorem.</li> <li>The <b>converse</b> of a statement is formed by reversing the order in which the statement is made. e.g. Statement: If P, then Q    Converse: If Q, then P.</li> <li>'<b>Implies</b>' is a term we can use in a proof when we write down a fact or conclusion that follows from previous statements. The symbol is for implies is: <math>\Rightarrow</math></li> </ol>

**3) Corollaries** (The 6 results below follow on from the 21 theorems above):

<p>1. A diagonal divides a parallelogram into 2 congruent triangles i.e. triangles A and B below are congruent.</p> 	<p>2. All angles at points of a circle, standing on the same arc, are equal, (and converse).</p>  <p style="text-align: right; border: 1px solid black; border-radius: 50%; padding: 5px; display: inline-block;">Watch for Butterfly Wings</p>
<p>3. Each angle in a semi-circle is a right angle.</p> 	<p>4. If the angle standing on a chord [BC] at some point, of the circle is a right-angle, then [BC] is a diameter.</p> 
<p>5. In a cyclic quadrilateral, then opposite angles sum to 180, (and converse).</p> <div style="display: flex; justify-content: space-around;">  <div style="border: 1px solid black; border-radius: 50%; padding: 10px; background-color: #ffffcc;"> <p style="text-align: center;"><math>A + C = 180^\circ</math> <math>B + D = 180^\circ</math></p> </div> </div>	<p>6. If two circles intersect at one point, their centres and the point of contact are collinear.</p>  <p style="text-align: right;">Tangent Point of contact</p>

**4) Constructions:**

<p><b>General Tips:</b></p> <ol style="list-style-type: none"> <li>1. Keep your work neat and tidy.</li> <li>2. Choose an appropriate pencil to draw the construction, not too dark and not too light.</li> <li>3. Draw rough sketches of construction first, especially for triangles and rectangles.</li> <li>4. Show all your construction lines &amp; label your construction.</li> </ol> <ul style="list-style-type: none"> <li>• There are 21 constructions on the course for Leaving Cert Ordinary Level. (See Booklet from class for step by step instructions)</li> </ul> <p><b>Constructions List:</b></p> <ol style="list-style-type: none"> <li>1. Bisector of a given angle, using only compass and straight edge.</li> <li>2. Perpendicular bisector of a segment, using only compass and straight edge.</li> <li>3. Line perpendicular to a given line l, passing through a given point not on l.</li> <li>4. Line perpendicular to a given line l, passing through a given point on l.</li> <li>5. Line parallel to a given line, through a given point.</li> </ol>	<ol style="list-style-type: none"> <li>6. Division of a line segment into 2 or 3 equal segments, without measuring it.</li> <li>7. Division of a line segment into any number of equal segments, without measuring it.</li> <li>8. Line segment of a given length on a given ray.</li> <li>9. Angle of a given number of degrees with a given ray as one arm.</li> <li>10 - 12. Triangle, given i) SSS ii) SAS or iii) ASA data</li> <li>13. Right-angled triangle, given the length of the hypotenuse and one other side.</li> <li>14. Right-angled triangle, given one side and one of the acute angles (several cases).</li> <li>15. Rectangle, given side lengths.</li> <li>16. Circumcentre and circumcircle of a given triangle, using ruler and compass.</li> <li>17. Incentre and incircle of a given triangle, using ruler and compass.</li> <li>18. Angle of <math>60^\circ</math>, without using a protractor or set square.</li> <li>19. Tangent to a given circle at a given point on it.</li> <li>20. Parallelogram, given the length of the sides and the measure of the angles.</li> <li>21. The centroid of a triangle.</li> </ol>
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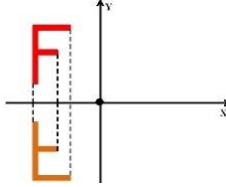
**5) Transformations/Symmetries/Enlargements:**

**a) Transformations:**

**Note:** In each of the pictures below, the red shape is the **object** and the second coloured shape is the **image**.

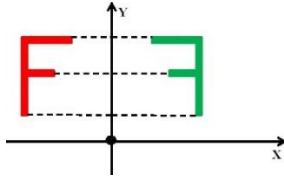
**Axial Symmetry in the X-axis: ( $S_x$ )**

- Shapes are mirrored/reflected in the X-axis. See example below.



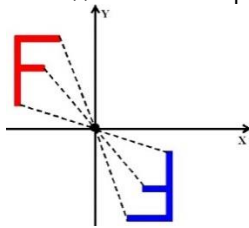
**Axial Symmetry in the Y-axis: ( $S_y$ )**

- Shapes are mirrored / reflected in the Y-axis. See example below.



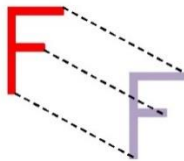
**Central Symmetry in the Origin: ( $S_o$ )**

- Shapes end up flipped and rotated as shown below.
- Central symmetry in a point other than the origin would have the same effect on the shape i.e. flipped and rotated



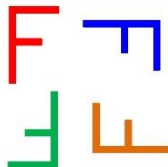
**Translation:**

- Note that shapes don't change when translated as the shape just 'slides' to another position



**Rotations:**

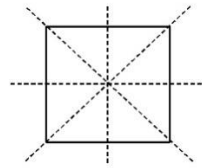
- The shape in blue below is a rotation of the red shape 90° clockwise. The green is a rotation of 180°. Note that it looks similar to the central symmetry in a point image from above. The orange is a rotation of 270° clockwise.



**b) Axes of Symmetries of Shapes:**

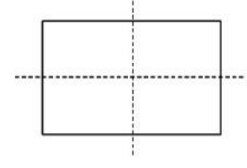
**Square:**

A square has 4 axes of symmetry, as shown below.



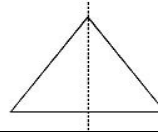
**Rectangle:**

A rectangle has 2 axes of symmetry, as shown below.



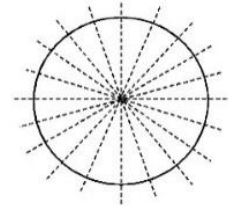
**Triangle:**

An isosceles triangle has 1 axis of symmetry, as shown below. If the triangle was an equilateral triangle it would have two more axes of symmetry from the other two vertices of the triangle.



**Circle:**

A circle has an infinite number of axes of symmetry, as shown below.



**Enlargements:**

**Notes:**

- An **enlargement** is a scaled up/down version of an object.
- The **scale factor 'k'** tells by how much the image has been scaled.
  - If  $k > 1 \Rightarrow$  scaled up
  - If  $0 < k < 1 \Rightarrow$  scaled down
- To find the scale factor from a given enlargement, divide a side of the image by its corresponding side in the object.
- The area of the image can be found by using:

$$\text{Area Image} = k^2 \times \text{Area of Object}$$

Not in Tables

- The **centre of enlargement** is the point where the object is being enlarged from.

**Steps for constructing an enlargement:**

- Using a ruler, draw dashed construction lines from the centre of enlargement  $o$ , out through some of the main points of the object.
- Measure the length of  $|OA|$ .
- Multiply  $|OA|$  by the scale factor  $k$  and then measure out that distance from  $o$  along the dashed line to find location of  $A'$ .
- Repeat for the other key points  $B, C, D, E$  etc.
- Join up  $A', B', C', D'$  etc. to form image.
- Check  $|A'B'| \div |AB|$  should be = the scale factor  $k$ .

**Example:** Enlargement for  $k = 3$  of small L shape is shown below.

