Week 30 Revision Sheet: Algebra 1 – 3 Worked Solutions

Q1.

$$45a^2 - 20$$
$$= 5(9a^2 - 4)$$
$$= 5\left[(3a)^2 - (2)^2\right]$$
$$= \boxed{5(3a+2)(3a-2)}$$

Q2.

$$\begin{array}{r} x^2 - x - 1 \\ 2x-3 \overline{)\,2x^3 - 5x^2 + x + 3} \\ (-)\;2x^3 \overset{(+)}{-} 3x^2 \\ \hline -2x^2 + x \\ (+)\;{-2x^2} \overset{(-)}{+} 3x \\ \hline -2x + 3 \\ (+)\;{-2x} \overset{(-)}{+} 3 \\ \hline 0 \end{array}$$

As Rem $= 0$

$\Rightarrow 2x-3$ is a factor

Q3.

$$3(x^2 - 9y^2)$$
$$3\left[(x)^2 - (3y)^2\right]$$
$$\boxed{3(x+3y)(x-3y)}$$

Q4.

$$5x^3 + 40y^3$$
$$= 5(x^3 + 8y^3)$$
$$= 5\left[(x)^3 + (2y)^3\right]$$
$$= \boxed{5(x+2y)(x^2 - 2xy + 4y^2)}$$

Q5.

$$\frac{x^2+4}{x^2-4} - \frac{x}{x+2}$$
$$= \frac{x^2+4}{(x-2)(x+2)} - \frac{x}{x+2}$$
$$= \frac{(x^2+4)(1) - x(x-2)}{(x-2)(x+2)}$$
$$= \frac{\cancel{x^2} + 4 - \cancel{x^2} + 2x}{(x-2)(x+2)}$$
$$= \frac{2x+4}{(x-2)(x+2)}$$
$$= \frac{2\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$$
$$= \boxed{\frac{2}{x-2}}$$

Q6.

$$\frac{5}{x^2-3x-10} - \frac{3}{x+2}$$
$$= \frac{5}{(x+2)(x-5)} - \frac{3}{x+2}$$
$$= \frac{5(1) - 3(x-5)}{(x+2)(x-5)}$$
$$= \frac{5 - 3x + 15}{(x+2)(x-5)}$$
$$= \boxed{\frac{20-3x}{(x+2)(x-5)}}$$

Q7.

$$\frac{x^2-2x-15}{x+5} \div \frac{x^2-9x+20}{3x+15}$$

Flip 2nd fraction & multiply:

$$= \frac{\cancel{(x-5)}(x+3)}{\cancel{x+5}} \times \frac{3\cancel{(x+5)}}{\cancel{(x-5)}(x-4)}$$
$$= \frac{3(x+3)}{x-4}$$
$$= \boxed{\frac{3x+9}{x-4}}$$

Q8.

$$\frac{x^2-36}{5x-10} \times \frac{x-2}{x+6}$$

$$\frac{(x-6)\cancel{(x+6)}}{5\cancel{(x-2)}} \times \frac{\cancel{x-2}}{\cancel{x+6}}$$

$$= \boxed{\frac{x-6}{5}}$$

Q9.

$$x^4 - y^4$$
$$= (x^2)^2 - (y^2)^2$$
$$= (x^2+y^2)(x^2-y^2)$$
$$= (x^2+y^2)\left((x)^2-(y)^2\right)$$
$$= \boxed{(x^2+y^2)(x+y)(x-y)}$$

Q10.

$$\begin{array}{r|l} & x^2-3x+2 \\ \hline x-2 & x^3-5x^2+8x-4 \\ & \underline{(-)\,x^3 \overset{(+)}{-} 2x^2} \\ & \quad -3x^2+8x \\ & \quad \underline{\overset{(+)}{-}3x^2 \overset{(-)}{+} 6x} \\ & \qquad 2x-4 \\ & \qquad \underline{(-)\,2x \overset{(+)}{-} 4} \\ & \qquad\quad 0 \end{array}$$

$$x^2-3x+2 = (x-2)(x-1)$$

$\Rightarrow$ Other factors: $\boxed{(x-2) \text{ and } (x-1)}$

Q11.

$$\frac{4x+6}{x^2+2x-35} \times \frac{x-5}{2x+3}$$
$$= \frac{2\cancel{(2x+3)}}{(x+7)\cancel{(x-5)}} \times \frac{\cancel{x-5}}{\cancel{2x+3}}$$
$$= \boxed{\frac{2}{x+7}}$$

Q12.

$$\frac{1}{x+1} + \frac{1}{x} = \frac{5}{6}$$
$$\frac{1(x)+1(x+1)}{(x+1)(x)} = \frac{5}{6}$$
$$\frac{x+x+1}{x^2+x} = \frac{5}{6}$$
$$\frac{2x+1}{x^2+x} = \frac{5}{6}$$
$$\Rightarrow 6(2x+1) = 5(x^2+x)$$
$$12x+6 = 5x^2+5x$$
$$5x^2-7x-6 = 0$$
$$(5x+3)(x-2) = 0$$
$$5x+3=0 \quad \text{or} \quad x-2=0$$
$$5x=-3 \qquad \boxed{x=2}$$
$$\boxed{x=-3/5}$$

Q13.

$$A: 3p+4q-2r=8$$
$$B: 9p+8q+2r=-13$$
$$C: 6p-12q+14r=-59$$

Solving A and B

$$A: 3p+4q-2r=8$$
$$B: 9p+8q+2r=-13$$
$$D: 12p+12q=-5$$

Solving B and C

$$B\times7: 9p+8q+14r=-91$$
$$C: -6p+12q-14r=59$$
$$E: 57p+68q=-32$$

Solving D and E

$$D\times17: 204p+204q=-85$$
$$E\times3: \overset{(-)}{-}171p \overset{(-)}{+} 204q = \overset{(+)}{-}96$$
$$33p = 11$$
$$p = 11/33 = \boxed{1/3}$$

Sub p into D

$$D: 12\left(\tfrac{1}{3}\right)+12q=-5$$
$$4+12q=-5$$
$$12q=-9$$
$$q=-9/12=\boxed{-3/4}$$

Sub p and q into A

$$A: 3\left(\tfrac{1}{3}\right)+4\left(\tfrac{-3}{4}\right)-2r=8$$
$$1-3-2r=8$$
$$-2r=10$$
$$r=\boxed{-5}$$

Q14.

$L: \ x - 4y = -13$

$C: \ x^2 + 2y^2 + 6xy = 29$

Using L

$x = 4y - 13 \quad *$

Sub * into C

$C: \ (4y-13)^2 + 2y^2 + 6(4y-13)y = 29$

$16y^2 + 169 - 104y + 2y^2 + 24y^2 - 78y - 29 = 0$

$42y^2 - 182y + 140 = 0$

$6y^2 - 26y + 20 = 0$

$3y^2 - 13y + 10 = 0$

$(3y - 10)(y - 1) = 0$

$3y - 10 = 0 \quad \text{or} \quad y - 1 = 0$

$3y = 10 \qquad y = 1$

$y = 10/3$

Using *

If $y = 1$

$x = 4(1) - 13$

$= -9$

If $y = 10/3$

$x = 4\left(\frac{10}{3}\right) - 13$

$= 1/3$

$\Rightarrow \boxed{\text{Ans: } (1/3, 10/3) \text{ or } (-9, 1)}$

Q15.

$x^3 + 5x^2 - 4x - 20 = 0$

Need to find a root first (which will be a factor of 20 $\Rightarrow$ try $\pm 1, \pm 2, \pm 4, \pm 5 \ldots$)

$f(1): \ (1)^3 + 5(1)^2 - 4(1) - 20 \neq 0$

$f(-1): \ (-1)^3 + 5(-1)^2 - 4(-1) - 20 \neq 0$

$f(2): \ (2)^3 + 5(2)^2 - 4(2) - 20 = 0$

$\Rightarrow x = 2$ is a root

$\Rightarrow x - 2$ is a factor

$$
\begin{array}{r l}
 & x^2 + 7x + 10 \\
x - 2 \,\big) & x^3 + 5x^2 - 4x - 20 \\
 & (-)\,x^3 \overset{(+)}{-} 2x^2 \\
 & \quad 7x^2 - 4x \\
 & (-)\,7x^2 \overset{(+)}{-} 14x \\
 & \quad\quad 10x - 20 \\
 & (-)\,10x \overset{(+)}{-} 20 \\
 & \quad\quad\quad 0
\end{array}
$$

$\Rightarrow x^3 + 5x^2 - 4x - 20 = 0$ becomes

$(x-2)(x^2 + 7x + 10) = 0$

$(x-2)(x+5)(x+2) = 0$

$x - 2 = 0 \qquad x + 5 = 0 \qquad x + 2 = 0$

$\boxed{x = 2} \qquad \boxed{x = -5} \qquad \boxed{x = -2}$

Q16.

Method 1:

$x = 5/2 \quad \text{or} \quad x = -3$

$2x = 5 \qquad x + 3 = 0$

$2x - 5 = 0$

$\Rightarrow (2x - 5)(x + 3) = 0$

$2x^2 - 5x + 6x - 15 = 0$

$\boxed{2x^2 + x - 15 = 0}$

Method 2:

$x^2 - x\left(\begin{smallmatrix}\text{SUM OF} \\ \text{ROOTS}\end{smallmatrix}\right) + \left(\begin{smallmatrix}\text{PRODUCT} \\ \text{OF ROOTS}\end{smallmatrix}\right) = 0$

$x^2 - x\left(\frac{5}{2} - 3\right) + \left(\frac{5}{2}\right)(-3) = 0$

$x^2 - x\left(-\frac{1}{2}\right) - \frac{15}{2} = 0$

$\boxed{2x^2 + x - 15 = 0}$

Q17.

Roots @ $x = -2, 0, 1, 3$

Triple root   Double root

$\Rightarrow (x+2)^3(x)(x-1)^2(x-3)$

Right hand tail is down and degree is odd

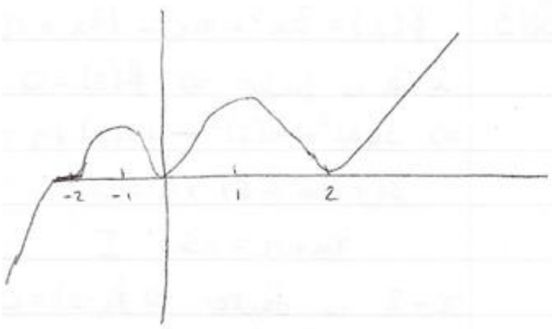
$\Rightarrow \ominus$ at start

$\Rightarrow \boxed{-(x+2)^3(x)(x-1)^2(x-3)}$

Q18.

Roots @ -2 (Triple), 0 (Double) and 2 (double) $\Rightarrow$ degree $= 7$

The 2 at the front will double the size of peaks and troughs but won't affect the roots (i.e. where the graph crosses the x-axis)



Q19.

$$
\begin{array}{r l}
 & x + (p-a) \\
x^2 + ax + 4 \,\big) & x^3 + px^2 + qx + 4b \\
(-) & x^3 + ax^2 + 4x \\
 & (p-a)x^2 + (q-4)x + 4b \\
(-) & (p-a)x^2 + a(p-a)x + 4(p-a) \\
 & (q-4-ap+a^2)x + (4b-4p+4a)
\end{array}
$$

x^2+ax+4 is a factor

$\Rightarrow q-4-ap+a^2=0$ and $4b-4p+4a=0$

$\Rightarrow b-p+a=0$

$\Rightarrow \boxed{p=a+b}$

Sub p into

$q-4-a(a+b)+a^2=0$

$q-4-\cancel{a^2}-ab+\cancel{a^2}=0$

$\boxed{q=4+ab}$

Q20.

$x^2+4x-6=(x+a)^2+b$

$x^2+4x-6=x^2+2ax+a^2+b$

$x^2+4x-6=x^2+2ax+(a^2+b)$

Comparing LHS and RHS

x terms must be equal

$\Rightarrow 4=2a$

$\Rightarrow \boxed{a=2}$

$-6=a^2+b$ also

$\Rightarrow -6=(2)^2+b$

$\Rightarrow \boxed{b=-10}$

Q21.

$f(x)=3x^3+mx^2-17x+n$

$x-3$ is factor $\Rightarrow f(3)=0$

$\Rightarrow 3(3)^3+m(3)^2-17(3)+n=0$

$81+9m-51+n=0$

$9m+n=-30$ I

$x+2$ is factor $\Rightarrow f(-2)=0$

$\Rightarrow 3(-2)^3+m(-2)^2-17(-2)+n=0$

$-24+4m+34+n=0$

$4m+n=-10$ II

Solving I and II

$9m+n=-30$

$(-)\ 4m+n=-10$

$5m=-20$

$\boxed{m=-4}$

Put m into I

$9m+n=-30$

$9(-4)+n=-30$

$-36+n=-30$

$\boxed{n=6}$

Q22.

$x^2-(3k+1)x+(2k^2+k+4)=0$

Real roots $\Rightarrow b^2-4ac \geq 0$

$\underline{b^2-4ac}$

$(3k+1)^2-4(1)(2k^2+k+4)\geq 0$

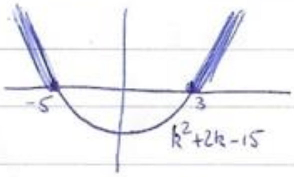
$9k^2+6k+1-8k^2-4k-16 \geq 0$

$k^2+2k-15 \geq 0$

Solve $k^2+2k-15=0$

$(k+5)(k-3)=0$

$k=-5 \quad k=3$



Using graph above

$k^2+2k-15 \geq 0$ when

$\boxed{k \leq -5}$ or $\boxed{k \geq 3}$

Q23.

$|2x-1|=7$

Method 1: (Defn. of modulus)

$2x-1=7 \quad$ or $\quad 2x-1=-7$

$2x=8 \qquad 2x=-6$

$\boxed{x=4} \qquad \boxed{x=-3}$

Method 2: (Sq both sides)

$(2x-1)^2=(7)^2$

$4x^2-4x+1=49$

$4x^2-4x-48=0$

$x^2-x-12=0$

$(x+3)(x-4)=0$

$\boxed{x=-3}$ or $\boxed{x=4}$

Q24.

(ii) $(a+b)\left(\frac{1}{a}+\frac{1}{b}\right)\geq 4$

$1+\frac{a}{b}+\frac{b}{a}+1 \geq 4$

Mult across by ab:

$ab+a^2+b^2+ab \geq 4ab$

$a^2+b^2+2ab-4ab \geq 0$

$a^2+b^2-2ab \geq 0$

$(a-b)(a-b)\geq 0$

$(a-b)^2 \geq 0$ which is true $\forall\, a,b>0$

$\Rightarrow a+b\left(\frac{1}{a}+\frac{1}{b}\right)\geq 4 \quad$ Q.E.D.

Q25.

$4ax^2-4ax+a+c^2=0$

No real roots $\Rightarrow b^2-4ac<0$

$\Rightarrow (-4a)^2-4(4a)(a+c^2) \leq 0$

$\cancel{16a^2}-\cancel{16a^2}-16ac^2 \leq 0$

$-16ac^2 \leq 0$

which is true $\forall\, a \in N$ and $c \in R$

Q26.

$\frac{x+3}{2x-1} \leq 4$

Multiply both sides by $(2x-1)^2$

$\Rightarrow (x+3)(2x-1) \leq 4(2x-1)^2$

$2x^2+6x-x-3 \leq 4(4x^2-4x+1)$

$2x^2+5x-3 \leq 16x^2-16x+4$

$\Rightarrow 14x^2-21x+7 \geq 0$

$2x^2-3x+1 \geq 0$

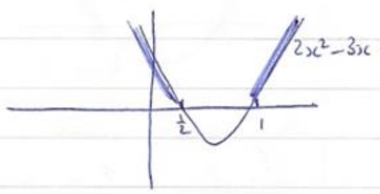
Solve $2x^2-3x+1=0$

$(2x-1)(x-1)=0$

$2x-1=0 \quad$ or $\quad x-1=0$

$2x=1 \qquad x=1$

$x=\frac{1}{2}$



From graph above $2x^2-3x+1 \geq 0$

when $\boxed{x \leq \frac{1}{2}}$ or $\boxed{x \geq 1}$

Q27.

i) $p^2 + 4q^2 \geq 4pq$

$p^2 - 4pq + 4q^2 \geq 0$

$(p - 2q)(p - 2q) \geq 0$

$(p - 2q)^2 \geq 0$

which is true $\forall\ p, q \in R$

$\Rightarrow p^2 + 4q^2 \geq 4pq$

Q.E.D.

ii) $(p+q)^2 \leq 2(p^2 + q^2)$

$p^2 + 2pq + q^2 \leq 2p^2 + 2q^2$

$-p^2 + 2pq - q^2 \leq 0$

$p^2 - 2pq + q^2 \geq 0$

$(p - q)(p - q) \geq 0$

$(p - q)^2 \geq 0$

which is true $\forall\ p, q \in R$

$\Rightarrow (p+q)^2 \leq 2(p^2 + q^2)$

Q.E.D.

Q28.

$|3x - 1| = |5x - 7|$

Method 1: (Defn of modulus)

$3x - 1 = 5x - 7$ or $3x - 1 = -5x + 7$

$-2x = -6$ $\qquad$ $8x = 8$

$\boxed{x = 3}$ $\qquad$ $\boxed{x = 1}$

Method 2: (Sq. both sides)

$(3x - 1)^2 = (5x - 7)^2$

$9x^2 - 6x + 1 = 25x^2 - 70x + 49$

$16x^2 - 64x + 48 = 0$

$x^2 - 4x + 3 = 0$

$\Rightarrow \boxed{x = 3}$ or $\boxed{x = 1}$

Q29.

$\sqrt{2x+5} - x = 1$

$\sqrt{2x+5} = 1 + x$

Sq. both sides:

$2x+5 = (1+x)^2$

$2x+5 = x^2 + 2x + 1$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x = \pm 2$

Checking both eliminates $x = -2$

$\Rightarrow \boxed{x = 2}$

Q30.

$\sqrt{6x+4} - 1 = \sqrt{3x+1}$

Sq. both sides:

$\left(\sqrt{6x+4} - 1\right)^2 = \left(\sqrt{3x+1}\right)^2$

$6x+4+1 - 2\sqrt{6x+4} = 3x+1$

$6x+5 - 2\sqrt{6x+4} = 3x+1$

$-2\sqrt{6x+4} = -3x - 4$

Sq. both sides again:

$\left(-2\sqrt{6x+4}\right)^2 = (-3x-4)^2$

$4(6x+4) = 9x^2 + 24x + 16$

$\cancel{24x} + 16 = 9x^2 + \cancel{24x} + 16$

$9x^2 = 0$

$x^2 = 0$

$\Rightarrow \boxed{x = 0}$