Topic 10: Trigonometry

1) The Basics:

a) Solving Problems:

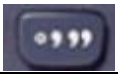
Steps when answering questions

1. Draw a good-sized diagram.
2. Fill in as much information as you can first e.g. the 3rd angle in a triangle where you're given the other 2 angles
3. Label what you're looking for.
4. Is there a right-angled triangle I can use?
   If Yes => Pythagoras Thm, SOHCAHTOA (Section 2 below)
   If No => Go to step 5
5. Do I know an angle and its opposite side?
   If Yes => Sine Rule (Section 3a below)
   If No => Cosine Rule (Section 3b below)

b) Calculator Use:

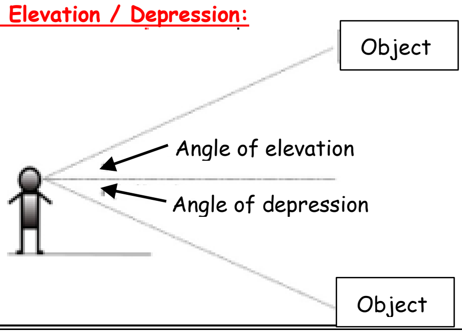
Notes:

- Make sure your calculator is in 'Degree' mode i.e. there is a 'DEG' or a 'D' on the top of your screen.
- If you know the angle, and you want to find Sin, Cos or Tan of it, you can just type it in straight.
  e.g. sin52 = $\boxed{\text{SIN}}$ $\boxed{52}$ $\boxed{=}$ 0.788
- When looking for an angle, then you need to use the SHIFT or 2ndF button in the top left corner of the calculator.
  e.g. Cos A = 0.4534
  => A = $\boxed{\text{SHIFT}}$ $\boxed{\text{COS}}$ $\boxed{0.4534}$ $\boxed{=}$ 63.04°
- To change between degrees and degrees and minutes as well. The button on the Casio calculator for doing that is:



Press this after getting the answer.

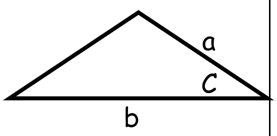
c) Angles of Elevation / Depression:



d) Clinometer

- We can measure angles of elevation / depression using a **clinometer**, as shown below:

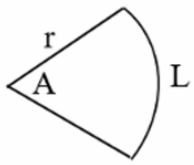
e) Area of a Triangle:



- Area can be found when we know **2 sides and the angle in between** the 2 sides:

$$Area = \frac{1}{2}abSinC$$

f) Sectors (See Area/Volume topic also)



$$Area = \frac{A}{360} X \pi r^2$$

$$L = \frac{A}{360} X 2\pi r$$

Tables pg 9

2) Right Angled Triangles:

a) Pythagoras' Theorem:

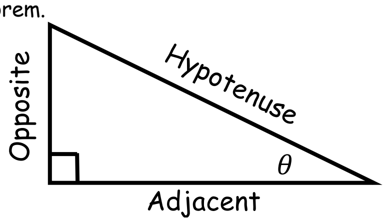
Notes:

- We can use **Pythagoras' Theorem** if we know two sides of a right-angled triangle and we want to find the third side i.e.

$$H^2 = O^2 + A^2$$

Tables pg 16

- Make sure and label the hypotenuse correctly when using this theorem.

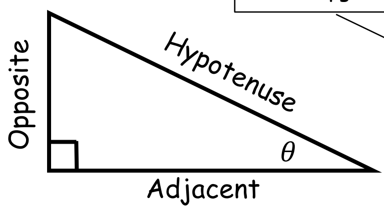


b) Sine, Cosine, Tan Ratios:

Notes:

- 'θ' is a Greek letter called 'theta'. It is often used to represent angles.
- Another way to remember the sin, cos and tan ratios is Silly Old Harry, Caught A Herring, Trawling Off America (SOHCAHTOA)

Tables pg 16



$$sin\theta = \frac{OPP}{HYP}$$

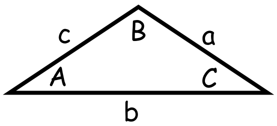
$$cos\theta = \frac{ADJ}{HYP}$$

$$tan\theta = \frac{OPP}{ADJ}$$

3) Non-Right Angled Triangles:

Sine Rule:

- Used if you know a side and its opposite angle
- Side 'a' must be across from angle 'A' and the same for 'b' and 'B'



$$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$$

Tables pg 16

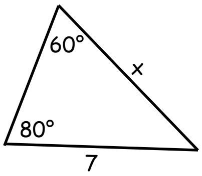
Or

$$\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$$

Not in Tables

Example:

Find x in the diagram below.



$$\frac{a}{SinA} = \frac{b}{SinB}$$

$$\frac{x}{Sin80} = \frac{7}{Sin60}$$

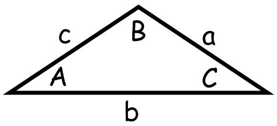
$x(Sin60) = 7(Sin80)$ (Cross Multiply)

=> $x = \frac{7(Sin80)}{Sin60}$ (÷ both sides by sin60)

=> $x = 7.96$

Cosine Rule:

- Used if Sine Rule can't be used
- The side you label 'a' **must** be across from the angle you label 'A'. Label the unknown side 'a' or label the unknown angle 'A'.



$$a^2 = b^2 + c^2 - 2bccosA$$

Tables pg 16

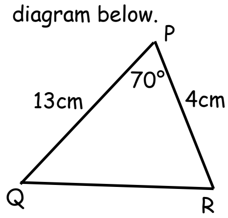
Or

$$cosA = \frac{b^2 + c^2 - a^2}{2bc}$$

Not in Tables

Example:

Find |QR| in the diagram below.



Label unknown side 'a'

=> 70 angle = 'A'

$$a^2 = b^2 + c^2 - 2bccosA$$

$$a^2 = (13)^2 + (4)^2 - 2(13)(4)ccos70$$

$$a^2 = 185 - 35.57$$

$$a^2 = 149.43$$

$$a = \sqrt{149.43}$$

$$a = 12.22$$

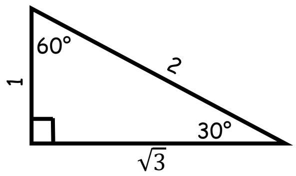
4) Special Angles/Unit Circle:

a) Special Angles:

- Use the table below (pg 13 of Tables) to write down the sin, cos or tan of the angles shown, in the form $\frac{a}{b}$

A (degrees)	0°	90°	180°	270°	30°	45°	60°
A (radians)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
cos A	1	0	-1	0	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
sin A	0	1	0	-1	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
tan A	0	-	0	-	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

- Useful to know the right-angled triangles these ratios come from. e.g.



Sin 30 = $\frac{OPP}{HYP} = \frac{1}{2}$

Cos 30 = $\frac{ADJ}{HYP} = \frac{\sqrt{3}}{2}$

Tan 60 = $\frac{OPP}{ADJ} = \frac{\sqrt{3}}{1}$

- Can also to simplify expressions into surd form

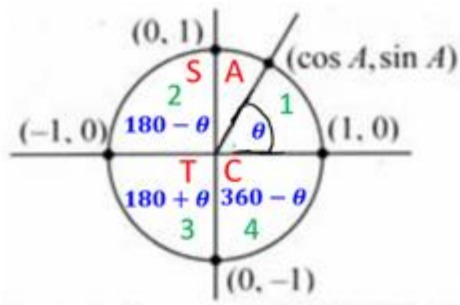
Example: Write $cos30 + sin30$ in surd form.

$$cos30 + sin60 = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$$

b) Unit Circle:

Notes:

- Need to be able to write sin, cos and tan of angles that are bigger than 90 in surd form, without a calculator.



Examples: Write i) sin 150 and ii) cos225 iii) sin 300 in surd form

i) 150 in quadrant 2 => will be positive for sin

Ref Angle = $180 - \theta = 150$ => $\theta = 30°$

=> sin150 = + sin30 = $+\frac{1}{2}$

ii) 225 in quadrant 3 => will be negative for cos

Ref Angle = $180 + \theta = 225$ => $\theta = 45°$

=> cos225 = - cos 45 = $-\frac{1}{\sqrt{2}}$

iii) 300 in quadrant 4 => will be negative for sin

Ref Angle = $360 - \theta = 300$ => $\theta = 60°$

=> sin300 = - sin60 = $-\frac{\sqrt{3}}{2}$