Dynamic Programming Examples:

a) Routing Problems:

Example 1: Sam has an ice-cream van from which she sells ice cream. She visits a different festival every week. She has to decide which three festivals to visit over the next three weeks. She starts the three-week period at home and finishes at home. She will spend one week at each of the three festivals she chooses, by travelling directly from one festival to the next. Table 1 gives the week in which 8 possible festivals will be held. Table 2 gives the expected profits from visiting each festival. Table 3 gives the cost of travel between festivals (and between home and festivals).

Table 1:

Week	1	2	3
Festivals	A,B,C	D,E	F,G,H

Table 2:

Festival	A	B	C	D	E	F	G	H
Expected Profit	900	800	1000	1500	1300	500	700	600

Table 3:

Travel Cost (€)	A	B	C	D	E	F	G	H
Home	100	80	150			80	90	70
A				180	150			
B				140	120			
C				200	210			
D						200	160	120
E						170	100	110

(i) Use Dynamic Programming to find the schedule that maximises the total expected profit, taking into account travel costs.

(ii) Which festivals should she visit and what is the maximum expected profit?

Solution:

Stage	State	Action	Destination	Value
# weeks to go	Where Sam visits	Where she travels	Where she finishes	= Profit – Travel + OVD

Optimal Path = _____ Festivals:_____ Overall Profit:_____

b) Stock Control Problems:

Example 2: Alex produces electric scooters. He can produce up to four a month, but if he wishes to produce more than three in any month, he will have to hire an assistant at a cost of €350 for that month. In any month when scooters are produced, the overheads (heating, electricity, lighting etc.) cost €200. A maximum of three scooters can be held in stock in any one month, at a cost of €40 per scooter per month. Scooters are delivered at the end of the month. The order book for scooters (all of which must be met on time) is:

Month	Aug	Sep	Oct	Nov
No. of Scooters Required	3	3	5	2

(i) What is the cost of storing 2 scooters and producing 4 scooters, in a given month?

(ii) If there is no stock at the beginning of August and Alex plans to have no stock in storage at the end of November, find the minimum cost for Alex to meet his orders and how he does it.

(iii) If the parts are imported from China at a cost of €600 per scooter and if Alex pays himself a salary of €1800 per month, and if he sells the scooters of €2000 each, find the overall profit over the 4-month period.

Solution:

Stage	State	Action	Destination	Value
The month (No. in order book)	No. in Stock	No. Made	No. Left in Stock	Costs = Storage + Overheads + Hired Labour + OVD

Optimum Costs = _____ Overall Profit = _____

c) Allocation of Resources:

Example 3: Sarah owns an apple orchard. Every year after the harvest, she sells the apples to a supermarket chain. If she has a particularly good crop, she uses the excess apples to make either Apple Sauce, Cider or feed for pigs. This year she has 5 tonnes left after delivering her orders to the supermarket chain. The price (in euro) she will get for these is as follows:

Tonnes	1	2	3	4	5
Feed for Pigs	85	165	245	325	405
Cider	165	185	205	225	245
Apple Sauce	125	275	335	365	385

Sarah will allocate her 5 tonnes in units of 1 tonne. She has already got an order for Apple Sauce, which will use 1 tonne of apples. (This means that she can use at most 4 tonnes for any other use)

(i) Which would get more profit: allocating 2 tonnes each to pig-feed and cider, along with one tonne to apple sauce OR 2 tonnes to cider and 3 to apple sauce?

(ii) Use dynamic programming to decide how Sarah should allocate her 5 tonnes of surplus apples in order to optimise her profits – and what is the maximum profit?

Solution:

(i) First option = _____ Second option = _____

Stage	State	Action	Destination	Value
(Product)	(Tonnes avail)	(Tonnes allocated)	(Tonnes remaining)	= profit + OVD
3				
(ap sauce)				
2				
(Cider)				
1				
(Pig Feed)				

Optimal Plan:_____ Profit = _____

d) Equipment Replacement and Maintenance:

Example 4:

A new van costs €25,000. Maintenance costs are as follows: €300 in the first year, €500 in the second year and €1000 in the third year (as tyres, batteries and brakes begin to wear). The resale value of a second-hand van is €20,000 after 1 year, €17,000 after 2 years and €15,000 after 3 years. A self-employed house painter buys a new van and wants to use dynamic programming to decide what is the best strategy over the next 6 years: how often should she replace her van over these 6 years? Assume that there is no re-sale value after 3 years (she must sell after 1, 2 or 3 years).

Solution:

Stage	State	Action	Destination	Value (Cost)
(Year)	(No. of yrs left when you buy this van)	(Decide to keep van for this no. of years)	(No. of yrs left when you sell the van)	= Cost of a new van + maintenance – resale value + OVD
0				
1				
2				
3				
4				
5				
6				

Best Strategy:_____ Optimal Costs:_____

Exercise 12I:

Q1.

Stage	State	Action	Destination	Value (Cost)

Q2.

Stage	State	Action	Destination	Value (Cost)

Q3.

Stage	State	Action	Destination	Value (Cost)

Q4.

Month	No. in Stock	No. Made	No. Left	Cost

Q5.

Month	No. in Stock	No. Made	No. Left	Cost

Q6.

Month	No. in Stock	No. Made	No. Left	Cost

Q7.

Product	Units Avail.	Units Alloc.	Units Left	Profit

Q9.

Subject	Days Avail.	Days Alloc.	Days Left	% Increase

Q8.

Product	Units Avail.	Units Alloc.	Units Left	Profit

Q10.

Year	No. Yrs Left	Keep For	Sell When	Cost

Q11.

Year	No. Yrs Left	Keep For	Sell When	Loss

Q12.

Year	No. Yrs Left	Keep For	Sell When	Loss

Summary of Dynamic Programming Table Setups:

Type 1: Routing Problems

Stage	State	Action	Destination	Value
# weeks to go	Where Sam visits	Where she travels	Where she finishes	= Profit – Travel + OVD

Type 2: Stock Control Problems

Stage (Demand)	State	Action	Destination	Value
The month (No. in order book)	No. in Stock	No. Made	No. Left in Stock	Costs = Storage + Overheads + Hired Labour + OVD
Nov (2)				

Type 3: Allocation of Resources

Stage	State	Action	Destination	Value
(Product)	(Tonnes available)	(Tonnes allocated)	(Tonnes remaining)	= profit + OVD
3				
(ap sauce)				

Type 4: Equipment Maintenance

Stage	State	Action	Destination	Value (Cost)
(Year)	(No. of yrs left when you buy this van)	(Decide to keep van for his no. of years)	(No. of yrs left when you sell the van)	= Cost of a new van + maintenance – resale value + OVD
0				