Revision Sheet 3: Factors and Quadratic Equations

Q1. Quadratic Equations (3rd Year Algebra – Unit 1)

a) Factorise the following:

i) $x^2 + 5x$ ii) $9x^2 - 25$ iii) $x^2 - 3x - 18$

iv) $49p^2 - 1$ v) $2pq + 3p + 2mq + 3m$ vi) $3x^2 + 17x - 6$

vii) $2ab - ac - 2bd + cd$ viii) $4x^2 + 4x - 15$ ix) $3x^2 - x$

b) Solve the following equations WITHOUT using the quadratic formula:

i) $x^2 - 7x + 12 = 0$ ii) $x^2 - 6x = 0$ iii) $x^2 - 25 = 0$

iv) $4x^2 - 12x + 5 = 0$ v) $5x^2 - 6x = 0$ vi) $9x^2 - 16 = 0$

vii) $(x + 3)^2 = (x + 1)(2x + 3)$ viii) $5x^2 - 13x = 6$ ix) $4x^2 = 1$

c) Solve the following equations, and give your answers correct to 2 decimal places.

i) $x^2 + 2x - 5 = 0$ ii) $3x^2 + 7x - 5 = 0$ iii) $2x^2 = 7x - 4$

d) Solve the following equations, and give your answers in surd form:

i) $3x^2 - 6x + 2 = 0$ ii) $4x^2 + 3x = 5$

e) i) What quadratic equation has roots -4 and 3 ?

ii) What quadratic equation has roots $^2/_3$ and $\frac{1}{5}$?

f) Factorise the following **fully**:

i) $p^3 - 25p$ ii) $4x^2 - 8x - 12$ iii) $3p^2 - 3p - 18$ iv) $8m^3 - 50m$

g) Simplify the following:

i) $\dfrac{8a + 8b}{a + b}$ ii) $\dfrac{a - 2}{a^2 + 5a - 14}$ iii) $\dfrac{x^2 + x - 30}{x - 5}$ iv) $\dfrac{ab - ac}{b - c}$

v) $\dfrac{a^2 b - a}{ab - a}$ vi) $\dfrac{5 - r}{r - 5}$ vii) $\dfrac{m - n}{3m - 3n}$ viii) $\dfrac{p^2 - q^2}{p^2 + 2pq + q^2}$

Q2. Extra Challenge Questions and Problem Solving:

a)   If $x = 3t - 1$ and $y = t + 7$, express $\dfrac{7x + y}{3y - x}$ in its simplest form in terms of t.

b)   Simplify $(3x + b)(6x - 2b) - (2y + b)(4y - 2b)$ and factorise fully the simplified expression.

c)   If $x = 2$ is a root of the equation $3x^2 - 4x + c = 0$, find the value of c.

d)   The equation $x^2 - 10x + d = 0$ has equal roots, find the roots.

e)   The area of a rectangle with side lengths $(3x + 1)$ and $(x + 5)$ is 80 m². Find the length of the rectangle.

f)   Simplify, and hence, factorise $(3x - 2y)^2 - y(5y - 12x)$

g)   If $x = 2$ is a root of the equation $x^2 - x + r = 0$, find the value of r.

h) A square has side lengths 2x. Find an expression for the length of the diagonal of the square.

i) Using factorising, evaluate $97^2 - 3^2$.

j) Show that there are no real values for x for which $x^2 + 4x + 5 = 0$.

k) Explain briefly what the value of $b^2 - 4ac$ tells you about the roots of a quadratic equation $ax^2 + b + c = 0$.

l) The edge of a cube measures $(x - 2)$ cm. Write down an expression in x for the surface area of the cube. If the surface area is $294cm^2$, find the value of x. Hence find the volume of the cube in cm^3.

m) Find the quadratic equation that has roots 3b and 5b.

n) Factorise fully $a^4 - b^4$.

o) Simplify $(2x - z)(6x + 3z) - (6a - 3z)(2a + z)$ and factorise fully the simplified expression.

p) A quadratic function is shown on the right hand side. It crosses the x-axis at -2 and -4. What is the quadratic equation represented in the diagram in the form $ax^2 + bx + c = 0$?
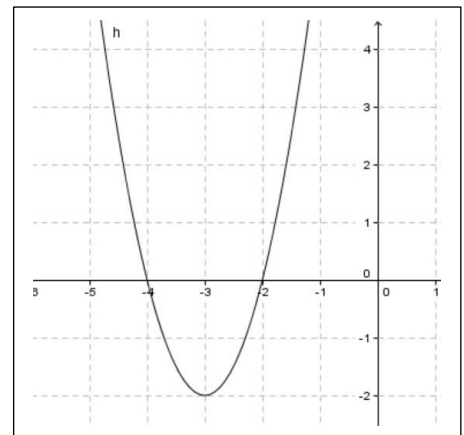
q) A rectangle has length 3cm greater than its width. If it has an area of $28cm^2$, find the length of the rectangle.

r) Given that $x^2 + y^2 + z^2 = 29$, find two possible values of y when x = 2 and z = -3.

s) The height h of a sliothar above the ground is given by the equation $h = 30t - 5t^2$, where h is the height in metres and t is the time in seconds since the ball was hit. After how many seconds is the ball at a height of:
    i) 0m above the ground
    ii) 40m above the ground?

t) The roots of the equation $x^2 + ax + b = 0$ are 2 and -1. Find the values of a and b.

<u>Revision Sheet 3 Solutions:</u>

<u>Q1.</u>

a)   i) $x(x + 5)$ ii) $(3x - 5)(3x + 5)$ iii) $(x - 6)(x + 3)$ iv) $(7p - 1)(7p + 1)$

      v) $(p + m)(2q + 3)$ vi) $(3x - 1)(x + 6)$ vii) $(a - d)(2b - c)$ viii) $(2x - 3)(2x + 5)$

      ix) $x(3x - 1)$

b)   i) $x = 3, x = 4$ ii) $x = 0, x = 6$ iii) $x = -5, x = 5$ iv) $x = \frac{1}{2}, x = \frac{5}{2}$

      v) $x = 0, x = \frac{6}{5}$ vi) $x = -\frac{4}{3}, x = \frac{4}{3}$ vii) $x = -2, x = 3$ viii) $x = -\frac{2}{5}, x = 3$

      ix) $x = -\frac{1}{2}, x = \frac{1}{2}$

c)   i) $x = 1.45, x = -3.45$ ii) $x = -2.9, x = 0.57$ iii) $x = 2.78, x = 0.72$

d)   i) $x = \frac{3 + \sqrt{3}}{3}, x = \frac{3 - \sqrt{3}}{3}$ ii) $x = \frac{-3 + \sqrt{89}}{8}, x = \frac{-3 - \sqrt{89}}{8}$

e)   i) $x^2 + x - 12 = 0$ ii) $15x^2 - 13x + 2 = 0$

f)   i) $p(p - 5)(p + 5)$ ii) $4(x - 3)(x + 1)$ iii) $3(p - 3)(p + 2)$ iv) $2(2m - 5)(2m + 5)$

g)   i) 8   ii) $\frac{1}{a + 7}$ iii) $x + 6$ iv) a

      v) $\frac{ab - 1}{b - 1}$ vi) -1   vii) $\frac{1}{3}$ viii) $\frac{p - q}{p + q}$

<u>Q2.</u>

a)   t   b) $18x^2 - 8y^2$, $2(3x - 2y)(3y + 2y)$ c) $x = -4$ d) $x = 5, x = 5$

e)   length = 10   f) $9x^2 - y^2$, $(3x - y)(3x + y)$ g) $r = -2$ h) diag $= \sqrt{8}x$

k) If $b^2 - 4ac > 0$, there will be 2 real roots. If $b^2 - 4ac = 0$, the roots will be equal. If $b^2 - 4ac < 0$, there are no real roots.

l) $6x^2 - 24x + 24$, $x = 9$, Volume = 343cm^3 m) $x^2 - 8bx + 15b^2 = 0$

n) $(a - b)(a + b)(a^2 + b^2)$ o) $12x^2 - 12a^2$, $12(x - a)(x + a)$ p) $x^2 + 6x + 8 = 0$

q) length = 7   r) $y = \pm 4$ s) i) 0 sec or 6 secs ii) 2 secs or 4 secs   t) $a = -1, b = -2$