Worked Solutions

Q1.

a) i) $x^4 \times x^3 = x^{4+3} = \boxed{x^7}$

ii) $\frac{a^8}{a^2} = a^{8-2} = \boxed{a^6}$

iii) $(y^2)^3 = \boxed{y^6}$

iv) $q^0 = \boxed{1}$

v) $16^{\frac{1}{2}} = \sqrt{16} = \boxed{4}$

vi) $2^{-3} = \frac{1}{2^3} = \boxed{\frac{1}{8}}$

b) i) $\sqrt{12} = \sqrt{4\times3} = \sqrt{4}\sqrt{3} = \boxed{2\sqrt{3}}$

ii) $\sqrt{50} = \sqrt{25\times2} = \sqrt{25}\sqrt{2} = \boxed{5\sqrt{2}}$

iii) $\sqrt{48} = \sqrt{16\times3} = \sqrt{16}\sqrt{3} = \boxed{4\sqrt{3}}$

iv) $\sqrt{200} = \sqrt{100\times2} = \boxed{10\sqrt{2}}$

c) i) $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = 2^2 = \boxed{4}$

ii) $16^{\frac{3}{4}} = \left(16^{\frac{1}{4}}\right)^3 = \left(\sqrt[4]{16}\right)^3 = 2^3 = \boxed{8}$

iii) $100^{\frac{3}{2}} = \left(100^{\frac{1}{2}}\right)^3 = \left(\sqrt{100}\right)^3 = 10^3 = \boxed{1000}$

iv) $64^{-\frac{1}{3}} = \frac{1}{64^{1/3}} = \frac{1}{\sqrt[3]{64}} = \boxed{\frac{1}{4}}$

v) $\left(\frac{9}{25}\right)^{-\frac{1}{2}} = \left(\frac{25}{9}\right)^{\frac{1}{2}} = \frac{\sqrt{25}}{\sqrt{9}} = \boxed{\frac{5}{3}}$

vi) $\left(\frac{27}{125}\right)^{-\frac{2}{3}} = \left(\frac{125}{27}\right)^{\frac{2}{3}} = \frac{125^{\frac{2}{3}}}{27^{\frac{2}{3}}}$

$= \frac{\left(125^{\frac{1}{3}}\right)^2}{\left(27^{\frac{1}{3}}\right)^2} = \frac{\left(\sqrt[3]{125}\right)^2}{\left(\sqrt[3]{27}\right)^2} = \left(\frac{5}{3}\right)^2 = \boxed{\frac{25}{9}}$

d) i) $\sqrt{80} + 2\sqrt{5}$

$= \sqrt{16\times5} + 2\sqrt{5}$

$= \sqrt{16}\sqrt{5} + 2\sqrt{5}$

$= 4\sqrt{5} + 2\sqrt{5}$

$= \boxed{6\sqrt{5}}$

ii) $\sqrt{20} + \sqrt{45} - 2\sqrt{5}$

$= \sqrt{5\times4} + \sqrt{5\times9} - 2\sqrt{5}$

$= \sqrt{5}\sqrt{4} + \sqrt{5}\sqrt{9} - 2\sqrt{5}$

$= 2\sqrt{5} + 3\sqrt{5} - 2\sqrt{5}$

$= \boxed{3\sqrt{5}}$

iii) $\sqrt{32} + 2\sqrt{8}$

$= \sqrt{16\times2} + 2\sqrt{4\times2}$

$= \sqrt{16}\sqrt{2} + 2\sqrt{4}\sqrt{2}$

$= 4\sqrt{2} + 4\sqrt{2}$

$= \boxed{8\sqrt{2}}$

e) i) $32 = \boxed{2^5}$ ii) $1 = \boxed{2^0}$

iii) $\frac{1}{8} = \frac{1}{2^3} = \boxed{2^{-3}}$

iv) $\sqrt{32} = \sqrt{2^5} = \left(2^5\right)^{\frac{1}{2}} = \boxed{2^{5/2}}$

v) $\frac{1}{\sqrt{8}} = \frac{1}{(2^3)^{1/2}} = \frac{1}{2^{3/2}} = \boxed{2^{-3/2}}$

vi) $8^{100} = (2^3)^{100} = \boxed{2^{300}}$

f) i) $9 = \boxed{3^2}$

ii) $\sqrt{27} = \sqrt{3^3} = (3^3)^{1/2} = \boxed{3^{3/2}}$

iii) $\frac{1}{9} = \frac{1}{3^2} = \boxed{3^{-2}}$

iv) $\frac{1}{\sqrt{27}} = \frac{1}{\sqrt{3^3}} = \frac{1}{3^{3/2}} = \boxed{3^{-\frac{3}{2}}}$

v) $\frac{81}{\sqrt{3}} = \frac{3^4}{3^{1/2}} = 3^{4-\frac{1}{2}} = \boxed{3^{\frac{7}{2}}}$

vi) $27^{40} = (3^3)^{40} = \boxed{3^{120}}$

g) i) $\sqrt{3}(\sqrt{3} - 2)$
$= (\sqrt{3})^2 - 2\sqrt{3}$
$= \boxed{3 - 2\sqrt{3}}$

ii) $\sqrt{2}(5\sqrt{2} - \sqrt{7})$
$= 5\sqrt{2}\sqrt{2} - \sqrt{2}\sqrt{7}$
$= 5(2) - \sqrt{14}$
$= \boxed{10 - \sqrt{14}}$

iii) $(\sqrt{3}+1)(\sqrt{3}-1)$
$= \sqrt{3}(\sqrt{3}-1) + 1(\sqrt{3}-1)$
$= (\sqrt{3})^2 - \cancel{\sqrt{3}} + \cancel{\sqrt{3}} - 1$
$= 3 - 1$
$= \boxed{2}$

OR Diff of 2 squares
$(\sqrt{3}+1)(\sqrt{3}-1)$
$= (\sqrt{3})^2 - (1)^2$
$= 3 - 1$
$= \boxed{2}$

h) i) $\frac{81^{\frac{1}{2}} \times 9^{-1}}{27^{2/3}}$
$= \frac{(3^4)^{1/2} \times (3^2)^{-1}}{(3^3)^{2/3}}$
$= \frac{3^2 \times 3^{-2}}{3^2}$
$= \frac{3^0}{3^2}$
$= 3^{0-2}$
$= \boxed{3^{-2}}$

ii) $\frac{3^3 \times 27^{2/3}}{3^{-3} \times 81^{3/4}}$
$= \frac{3^3 \times (3^3)^{2/3}}{3^{-3} \times (3^4)^{3/4}}$
$= \frac{3^3 \times 3^2}{3^{-3} \times 3^3}$
$= \frac{3^{3+2}}{3^{-3+3}}$
$= \frac{3^5}{3^0}$
$= 3^{5-0}$
$= \boxed{3^5}$

iii) $\dfrac{4^2 \times 16^{\frac{1}{2}}}{4^3 \times 64^{2/3}}$

$= \dfrac{4^2 \times (4^2)^{\frac{1}{2}}}{4^3 \times (4^3)^{2/3}}$

$= \dfrac{4^2 \times 4^1}{4^3 \times 4^2}$

$= \dfrac{4^{2+1}}{4^{3+2}}$

$= \dfrac{4^3}{4^5}$

$= 4^{3-5}$

$= \boxed{4^{-2}}$

i) i) $(1 - 2\sqrt{3})(1 + 2\sqrt{3})$

$= 1(1+2\sqrt{3}) - 2\sqrt{3}(1 + 2\sqrt{3})$

$= 1 + 2\sqrt{3} - 2\sqrt{3} - 4(3)$

$= 1 - 12$

$= \boxed{-11}$

OR Diff of 2 squares

$(1 - 2\sqrt{3})(1 + 2\sqrt{3})$

$= (1)^2 - (2\sqrt{3})^2$

$= 1 - 4(3)$

$= 1 - 12$

$= \boxed{-11}$

ii) $(3 - \sqrt{2})^2$

$= (3 - \sqrt{2})(3 - \sqrt{2})$

$= 3(3 - \sqrt{2}) - \sqrt{2}(3 - \sqrt{2})$

$= 9 - 3\sqrt{2} - 3\sqrt{2} + 2$

$= \boxed{11 - 6\sqrt{2}}$

j) $\dfrac{2\sqrt{45}}{\sqrt{10}} = \dfrac{2\sqrt{9 \times 5}}{\sqrt{5 \times 2}}$

$= \dfrac{2\sqrt{9}\cancel{\sqrt{5}}}{\cancel{\sqrt{5}}\sqrt{2}}$

$= \dfrac{2(3)}{\sqrt{2}}$

$= \dfrac{6}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{6\sqrt{2}}{2}$

$= \boxed{3\sqrt{2}}$

Q2 a) i) $3^x = 81$

$3^x = 3^4$

$\Rightarrow \boxed{x = 4}$

ii) $4^x = 32$

$(2^2)^x = 2^5$

$2^{2x} = 2^5$

$\Rightarrow \frac{2x}{2} = \frac{5}{2}$

$\boxed{x = \frac{5}{2}}$

iii) $9^x = 27$

$(3^2)^x = 3^3$

$3^{2x} = 3^3$

$\Rightarrow \frac{2x}{2} = \frac{3}{2}$

$\boxed{x = \frac{3}{2}}$

iv) $16^x = 32$

$(2^4)^x = 2^5$

$2^{4x} = 2^5$

$\Rightarrow \frac{4x}{4} = \frac{5}{4}$

$\boxed{x = \frac{5}{4}}$

v) $2^x = \frac{1}{8}$

$2^x = \frac{1}{2^3}$

$2^x = 2^{-3}$

$\Rightarrow \boxed{x = -3}$

vi) $5^x = \frac{1}{125}$

$5^x = \frac{1}{5^3}$

$5^x = 5^{-3}$

$\Rightarrow \boxed{x = -3}$

b) i) $27^x = 81$

$(3^3)^x = 3^4$

$3^{3x} = 3^4$

$\Rightarrow \frac{3x}{3} = \frac{4}{3}$

$\boxed{x = 4/3}$

ii) $2^{-x} = 16$

$2^{-x} = 2^4$

$\Rightarrow -x = 4$

$\boxed{x = -4}$

iii) $3^{x+2} = 81$

$3^{x+2} = 3^4$

$\Rightarrow x + 2 = 4$

$x = 4 - 2$

$\boxed{x = 2}$

iv) $2^x = \frac{\sqrt{2}}{8}$

$2^x = \frac{2^{1/2}}{2^3}$

$2^x = 2^{1/2 - 3}$

$2^x = 2^{-5/2}$

$\Rightarrow x = \boxed{-5/2}$

c) i) $2^{2x-2} = \frac{16}{\sqrt{8}}$

$2^{2x-2} = \frac{2^4}{(2^3)^{1/2}}$

$2^{2x-2} = \frac{2^4}{2^{3/2}}$

$2^{2x-2} = 2^{4-3/2}$

$2^{2x-2} = 2^{5/2}$

$\Rightarrow 2x - 2 = 5/2$

$2x = 5/2 + 2$

$\frac{2x}{2} = \frac{9/2}{2}$

$\Rightarrow x = \boxed{\frac{9}{4}}$

ii) $9^{3-x} = \frac{\sqrt{27}}{81}$

$(3^2)^{3-x} = \frac{(3^3)^{1/2}}{3^4}$

$3^{6-2x} = \frac{3^{3/2}}{3^4}$

$3^{6-2x} = 3^{3/2-4}$

$3^{6-2x} = 3^{-5/2}$

$\Rightarrow 6 - 2x = -\frac{5}{2}$

$2(6-2x) = -5$

$12 - 4x = -5$

$-4x = -5 - 12$

$\frac{-4x}{-4} = \frac{-17}{-4}$

$x = \boxed{\frac{17}{4}}$

iii) $16^{x-1} = 2\sqrt{32}$

$(2^4)^{x-1} = 2(2^5)^{1/2}$

$2^{4x-4} = 2(2^{5/2})$

$2^{4x-4} = 2^{1+5/2}$

$2^{4x-4} = 2^{7/2}$

$\Rightarrow 4x - 4 = \frac{7}{2}$

$2(4x-4) = 7$

$8x - 8 = 7$

$8x = 7 + 8$

$\frac{8x}{8} = \frac{15}{8}$

$x = \boxed{\frac{15}{8}}$

iv) $8^{3x} = 16^{30}$

$(2^3)^{3x} = (2^4)^{30}$

$2^{9x} = 2^{120}$

$\Rightarrow \frac{9x}{9} = \frac{120}{9}$

$x = \frac{120}{9} = \boxed{\frac{40}{3}}$

Q3.

a) $\left(\frac{27}{8}\right)^{\frac{1}{3}} + \left(\frac{100}{9}\right)^{-\frac{1}{2}}$

$= \frac{27^{\frac{1}{3}}}{8^{\frac{1}{3}}} + \left(\frac{9}{100}\right)^{\frac{1}{2}}$

$= \frac{\sqrt[3]{27}}{\sqrt[3]{8}} + \frac{\sqrt{9}}{\sqrt{100}}$

$= \frac{3}{2} + \frac{3}{10}$

$= \frac{15}{10} + \frac{3}{10}$

$= \frac{18}{10} = \boxed{\frac{9}{5}}$

b) $2^{10} + 2^{10} + 2^{10} + 2^{10}$

$= 4(2^{10})$

$= 2^2(2^{10})$

$= 2^{2+10}$

$= \boxed{2^{12}}$

c) $p^2 - q^2$

$= (\sqrt{5}+\sqrt{2})^2 - (\sqrt{5}-\sqrt{2})^2$

$= 5+2+\sqrt{10} - [5+2-\sqrt{10}]$

$= 7 + \sqrt{10} - 7 + \sqrt{10}$

$= \boxed{2\sqrt{10}}$

d) $\left(\sqrt{x} + \frac{2}{\sqrt{x}}\right)\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)$

$= \sqrt{x}\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right) + \frac{2}{\sqrt{x}}\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)$

$= x - \cancel{2} + \cancel{2} - \frac{4}{x}$

$= x - \frac{4}{x} \Rightarrow \boxed{p = 4}$

e) $b^{\frac{1}{2}}\left(b^{\frac{1}{2}} + b^{-\frac{1}{2}}\right)$

$= b + b^0$

$= \boxed{b + 1}$

f) $t = k\sqrt{x+5}$

$\frac{1}{4} = k\sqrt{\frac{5}{4}+5}$

$\frac{1}{4} = k\sqrt{\frac{25}{4}}$

$\frac{1}{4} = k\left(\frac{5}{2}\right)$

$\frac{1}{4} = \frac{5k}{2}$

$4(5k) = 1(2)$

$\frac{20k}{20} = \frac{2}{20}$

$\boxed{k = \frac{1}{10}}$

g) $x = \sqrt{a^2+b^2}$

$x = \sqrt{(\sqrt{2})^2 + (\sqrt{7})^2}$

$= \sqrt{2+7}$

$= \sqrt{9}$

$= \boxed{3}$