Worked Solutions

Q1. <u>LCM</u>

$$(0.2)(200) + (2)(0) = (0.2)(120) + (2)(v)$$

$$40 = 24 + 2v$$

$$2v = 16$$

$$v = \boxed{8\text{ m/s}}$$

Q2.

i)

Before	Mass	After
$11\vec{i}$	4	$p\vec{i}$
$7\vec{i}$	6	$10\vec{i}$

<u>NLR</u>

$$\frac{p-10}{11-7} = -e$$

$$p - 10 = -4e \quad (*)$$

<u>LCM</u>

$$11(4) + 7(6) = p(4) + 10(6)$$

$$4p + 60 = 86$$

$$4p = 26$$

$$p = 13/2$$

$\Rightarrow$ Speed : $\boxed{13/2\text{ m/s}}$ or $\boxed{6.5\text{ m/s}}$

ii) <u>Using (*)</u>

$$p - 10 = -4e$$

$$13/2 - 10 = -4e$$

$$-4e = -7/2$$

$$\Rightarrow e = \boxed{7/8}$$

Q3.

Before	Mass	After
$16\vec{i}$	1	$p\vec{i}$
$-9\vec{i}$	2	$q\vec{i}$

$e = 5/7$

NLR

$$\frac{p-q}{16+9} = \frac{-5}{7}$$

$$7p - 7q = -125 \quad : \mathrm{I}$$

LCM

$$16(1) + (-9)(2) = p(1) + q(2)$$

$$p + 2q = -2 \quad : \mathrm{II}$$

Solving I & II:

$$\cancel{7p} - 7q = -125$$

$$(-)\ \cancel{7p} \overset{(-)}{+} 14q = \overset{(+)}{-}14$$

$$-21q = -111$$

$$q = 37/7$$

$\Rightarrow$ II: $p + 2(37/7) = -2$

$$p + 74/7 = -2$$

$$p = -2 - 74/7$$

$$p = -88/7$$

$$KE_{loss} = KE_{Bef} - KE_{Aft}$$

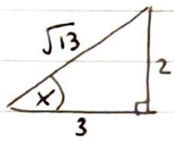
$$= \left[\tfrac{1}{2}(1)(16)^2 + \tfrac{1}{2}(2)(9)^2\right] - \left[\tfrac{1}{2}(1)\left(\tfrac{88}{7}\right)^2 + \tfrac{1}{2}(2)\left(\tfrac{37}{7}\right)^2\right]$$

$$= 209 - 106.959$$

$$= \boxed{102\ \mathrm{J}}$$

Q4.

Before	Mass	After
$3\vec{i} + 2\vec{j}$	M	$p\vec{i} + 2\vec{j}$
$0\vec{i} + 0\vec{j}$	$2M$	$q\vec{i} + 0\vec{j}$



$\cos x = \frac{3}{\sqrt{13}}$

$\sin x = \frac{2}{\sqrt{13}}$

i) NLR

$\frac{p - q}{3} = -\frac{1}{2}$

$2p - 2q = -3 \quad : \mathrm{I}$

LCM

$3(M) + 0 = p(M) + q(2M)$

$p + 2q = 3 \quad \mathrm{II}$

Solving I & II:

$\mathrm{I}: \quad 2p - \cancel{2q} = -3$

$\mathrm{II}: \quad p + \cancel{2q} = 3$

$3p \quad = 0$

$p = 0 \quad \Rightarrow \quad q = \frac{3}{2}$

$\Rightarrow$ Ans: $\boxed{0\vec{i} + 2\vec{j}, \quad \frac{3}{2}\vec{i} + 0\vec{j}}$

ii) $KE_{Bef} = \frac{1}{2}(M)(3)^2 = \frac{9M}{2}$

$KE_{Aft} = \frac{1}{2}(M)(0)^2 + \frac{1}{2}(2M)\left(\frac{3}{2}\right)^2 = \frac{9M}{4}$

$\Rightarrow$ Loss $= \frac{18M}{4} - \frac{9M}{4}$

$= \boxed{\frac{9M}{4} \text{ J}}$

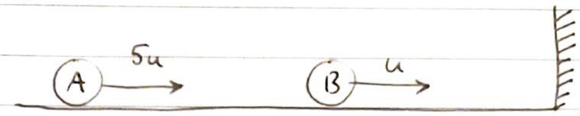
iii) $I = m\vec{v} - m\vec{u}$ (for M)

$= M(0\vec{i}) - M(3\vec{i})$

$= -3M\vec{i}$

$\Rightarrow$ Ans: $\boxed{3M \text{ Ns}}$

Q5.



i) $A \rightarrow B$ $\quad e = 3/4$

Before	Mass	After
$5u\vec{i}$	m	$p\vec{i}$
$u\vec{i}$	$6m$	$q\vec{i}$

NLR

$$\frac{p-q}{4u} = \frac{-3}{4}$$

$$4p - 4q = -12u$$

$$p - q = -3u : \mathrm{I}$$

LCM

$$5u(m) + u(6m) = p(m) + q(6m)$$

$$p + 6q = 11u : \mathrm{II}$$

Solving I & II:

$$\cancel{p} - q = -3u$$

$$(-)\cancel{p} \overset{(-)}{+} 6q = {}^{(-)}11u$$

$$-7q = -14u$$

$$q = 2u \Rightarrow \mathrm{I}: p - 2u = -3u$$

$$p = -u$$

$$\Rightarrow \text{Ans}: \boxed{-u\vec{i}, 2u\vec{i}}$$

ii) $B \rightarrow$ Wall $\quad e = 1/4$

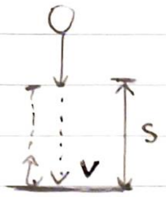
Vel before hits wall $= 2u\vec{i}$

$\Rightarrow$ Vel after hits wall $= -\frac{1}{4}(2u)\vec{i} = \frac{-u}{2}\vec{i}$

As $p < -u$ and $\frac{-u}{2} < 0$, both spheres are now moving to the left and speed of B is half of speed of A $\Rightarrow$ no more collisions occur.

Q6.

i)



$u = 0 \quad a = g \quad s = 3.5 \quad v = ?$

$v^2 = u^2 + 2as$

$v^2 = 0 + 2(g)(3.5)$

$v^2 = 68.6$

$v = \sqrt{68.6} = \boxed{8.29 \text{ m/s}}$

ii) Vel before impact $= -8.29\vec{j}$

$\Rightarrow$ Vel after impact $= +0.6(8.29)$

$= 4.97$

$u = 4.97 \quad v = 0 \quad a = -g \quad s = s$

$v^2 = u^2 + 2as$

$0 = (4.97)^2 + 2(-g)(s)$

$2gs = 24.7$

$s = \frac{24.7}{2g} = \boxed{1.26 \text{ m}}$

Q7.

Before	Mass	After
$4\vec{i}$	2	$p\vec{i}$
$-2\vec{i}$	5	$0\vec{i}$

i) NLR

$\frac{p - 0}{4+2} = -e$

$p = -6e$

LCM

$4(2) + (-2)(5) = p(2) + 0$

$2p = -2$

$\boxed{p = -1}$

Ans : $\boxed{1 \text{ m/s}}$

ii) $p = -6e$

$\Rightarrow -1 = -6e$

$\Rightarrow e = \boxed{1/6}$

iii) Loss in KE $= KE_{Bef} - KE_{Aft}$

$= \left[\frac{1}{2}(2)(4)^2 + \frac{1}{2}(5)(2)^2\right] - \left[\frac{1}{2}(2)(1)^2 + 0\right]$

$= 26 - 1$

$= 25 \text{ J}$

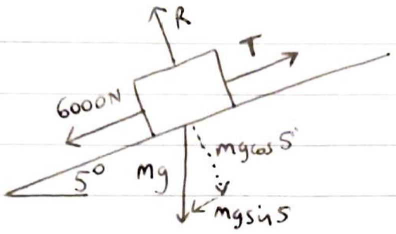
$\Rightarrow$ % Loss $= \frac{\text{Loss}}{\text{Before}} \times \frac{100}{1}$

$= \frac{25}{26} \times \frac{100}{1}$

$= \boxed{96\%}$

Q8.

i)



$P = Tv$

$400\,000 = Tv$

$\Rightarrow v = \dfrac{400\,000}{T}$

<u>Forces up hill</u> (Max Speed $\Rightarrow a = 0$)

$T = 6000 + mg\sin 5$

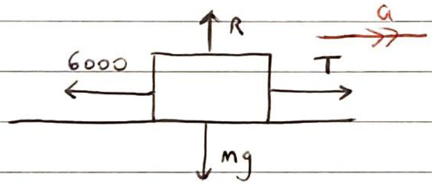
$T = 6000 + 100\,000g\sin 5$

$T = 91412.63$

$T = 91413\text{ N}$

$\Rightarrow$ Max Speed $= \dfrac{400000}{91413} = \boxed{4.38\text{ m/s}}$

ii)



$T - 6000 = 100000a$

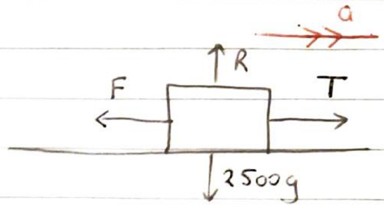
$91413 - 6000 = 100000a$

$100000a = 85413$

$a = \dfrac{85413}{100000} = \boxed{0.85\text{ m/s}^2}$

Q9.

i)



$$\text{Power} = Tv$$

$$21000 = T(15)$$

$$\Rightarrow T = \frac{21000}{15} = 1400\,N$$

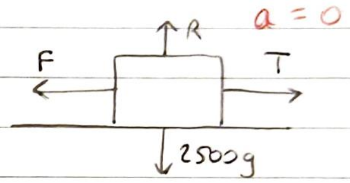
$$F = ma$$

$$1400 - 400 = 2500a$$

$$1000 = 2500a$$

$$\Rightarrow a = \frac{1000}{2500} = \boxed{\tfrac{2}{5}\ m/s^2} \quad \text{or} \quad \boxed{0.4\ m/s^2}$$

ii) Max Speed $\Rightarrow a = 0$



$$\text{Power} = Tv$$

$$21000 = Tv$$

$$\Rightarrow T = \frac{21000}{v}$$

$$a = 0 \Rightarrow \frac{21000}{v} = F = 400$$

$$\Rightarrow 400v = 21000$$

$$\Rightarrow v = \frac{21000}{400} = \boxed{52.5\,m/s}$$

Q10.

Before	Mass	After
$u\cos 30\vec{i} + u\sin 30\vec{j}$	1	$p\vec{i} + u\sin 30\vec{j}$
$0\vec{i} + 0\vec{j}$	2	$q\vec{i} + 0\vec{j}$

$u\cos 30 = \frac{u\sqrt{3}}{2}$

$u\sin 30 = \frac{u}{2}$

Perfectly elastic $\Rightarrow$ $e = 1$

NLR

$$\frac{p - q}{\frac{u\sqrt{3}}{2}} = -1$$

$$2p - 2q = -u\sqrt{3} \quad : \mathrm{I}$$

LCM

$$\frac{u\sqrt{3}}{2}(1) + 0 = p(1) + q(2)$$

$$p + 2q = \frac{u\sqrt{3}}{2}$$

$$2p + 4q = u\sqrt{3} \quad \mathrm{II}$$

Solving I & II:

$\mathrm{I}\times 2: \quad 4p - \cancel{4q} = -2u\sqrt{3}$

$\mathrm{II}: \quad 2p + \cancel{4q} = u\sqrt{3}$

$6p = -u\sqrt{3}$

$p = -u\sqrt{3}/6$

Solving I & II:

$\mathrm{I}: \quad \cancel{2p} - 2q = -u\sqrt{3}$

$-\mathrm{II}: \quad \cancel{-2p} - 4q = -u\sqrt{3}$

$-6q = -2u\sqrt{3}$

$q = \frac{u\sqrt{3}}{3}$

Vel 1kg After $= \left| -\frac{u\sqrt{3}}{6}\vec{i} + \frac{u}{2}\vec{j} \right|$

$$= \sqrt{\left(-\frac{u\sqrt{3}}{6}\right)^2 + \left(\frac{u}{2}\right)^2}$$

$$= \sqrt{\frac{3u^2}{36} + \frac{u^2}{4}}$$

$$= \sqrt{\frac{3u^2}{36} + \frac{9u^2}{36}}$$

$$= \sqrt{\frac{12u^2}{36}}$$

$$= \boxed{\frac{u\sqrt{3}}{3} = q}$$

Q.E.D.

Q11.

Before	Mass	After
$2u\vec{i}$	m	$p\vec{i}$
$u\vec{i}$	$2m$	$q\vec{i}$

i) NLR

$$\frac{p-q}{u} = -e$$

$$p-q = -eu \quad : \mathrm{I}$$

LCM

$$2u(m) + u(2m) = p(m) + q(2m)$$

$$p + 2q = 4u \quad \mathrm{II}$$

Solving I & II:

$$\cancel{p} - q = -eu$$

$$(-)\ \cancel{p} \overset{(-)}{+} 2q \overset{(-)}{=} 4u$$

$$-3q = -eu - 4u$$

$$3q = eu + 4u$$

$$q = \frac{u(e+4)}{3}$$

Solving I & II:

$$\mathrm{I}\times 2: \quad 2p - \cancel{2q} = -2eu$$

$$\mathrm{II}: \quad p + \cancel{2q} = 4u$$

$$3p = 4u - 2eu$$

$$p = \frac{2u(2-e)}{3} \quad *$$

As $0 \leq e \leq 1$, $\frac{e+4}{3}$ will be $> 1 \Rightarrow$ velocity of 2^{nd} sphere increases.

ii) $p = u$

$$\Rightarrow \frac{u}{1} = \frac{2u(2-e)}{3} \quad (\text{Using } *)$$

$$\Rightarrow 3\cancel{u} = 2\cancel{u}(2-e)$$

$$3 = 4 - 2e$$

$$2e = 1$$

$$e = \boxed{1/2}$$

Q12.

Before	Mass	After
$u\cos\alpha\,\vec{i} + u\sin\alpha\,\vec{j}$	m	$p\vec{i} + u\sin\alpha\,\vec{j}$
$0\vec{i} + 0\vec{j}$	m	$q\vec{i} + 0\vec{j}$

NLR

$$\frac{p-q}{u\cos\alpha} = -e$$

$$p - q = -eu\cos\alpha \quad : \mathrm{I}$$

LCM

$$u\cos\alpha(m) + 0 = p(m) + q(m)$$

$$p + q = u\cos\alpha \quad : \mathrm{II}$$

Solving I & II:

$$p - \cancel{q} = -eu\cos\alpha$$
$$p + \cancel{q} = u\cos\alpha$$
$$2p = u\cos\alpha(1-e)$$
$$p = \frac{u\cos\alpha(1-e)}{2}$$

$$\text{Slope before} = \frac{\vec{j}\text{ comp}}{\vec{i}\text{ comp}} = \frac{u\sin\alpha}{u\cos\alpha} = \tan\alpha$$

$$\text{Slope after} = \frac{u\sin\alpha}{\frac{u\cos\alpha(1-e)}{2}} = \frac{2\tan\alpha}{1-e}$$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\tan\alpha - \frac{2\tan\alpha}{1-e}}{1 + \tan\alpha\left(\frac{2\tan\alpha}{1-e}\right)}\right| \times \frac{1-e}{1-e}$$

$$= \left|\frac{(1-e)\tan\alpha - 2\tan\alpha}{(1-e) + 2\tan^2\alpha}\right|$$

$$= \left|\frac{-\tan\alpha(1+e)}{1-e+2\tan^2\alpha}\right|$$

$$= \boxed{\frac{\tan\alpha(1+e)}{(1-e)+2\tan^2\alpha}} \qquad \text{Q.E.D.}$$

Q13.

Before	Mass	After
$u\vec{i}$	km	$0\vec{i}$
$ku\vec{i}$	m	$p\vec{i}$

i)

<u>NLR</u>

$$\frac{0-p}{u-ku} = -e$$

$$-p = -eu + eku$$

$$p = eu - eku$$

$$p = eu(1-k)$$

<u>LCM</u>

$$u(km) + ku(m) = 0 + p(m)$$

$$p = 2uk$$

Ans : $\boxed{2uk \text{ m/s}}$

ii) $2\cancel{u}k = e\cancel{u}(1-k)$

$$\Rightarrow e = \frac{2k}{1-k}$$

$$0 < e \leq 1 \Rightarrow \frac{2k}{1-k} \leq 1$$

$$\frac{2k}{1-k}(1-k)^2 \leq 1(1-k)^2$$

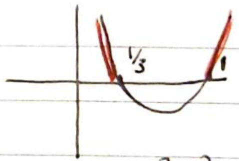
$$2k(1-k) \leq k^2-2k+1$$

$$2k-2k^2-k^2+2k-1 \leq 0$$

$$-3k^2+4k-1 \leq 0$$

$$3k^2-4k+1 \geq 0$$

$$(3k-1)(k-1)=0$$



$$\Rightarrow k \leq \tfrac{1}{3} \text{ or } \underbrace{k > 1}_{\text{This makes } e<0} \quad (\text{If } k=1 \; e = 2/0)$$

$\Rightarrow$ exclude

$$\Rightarrow \boxed{k \leq \tfrac{1}{3}}$$

Q14.

Before	Mass	After
$3u\cos 30\vec{i} + 3u\sin 30\vec{j}$	$3m$	$0\vec{i} + 3u\sin 30\vec{j}$
$-2u\vec{i} + 0\vec{j}$	$2m$	$p\vec{i} + 0\vec{j}$

$\cos 30 = \sqrt{3}/2$

$\sin 30 = 1/2$

NLR

$$\frac{0 - p}{3u\cos 30 + 2u} = -e$$

$$\frac{-p}{3u(\sqrt{3}/2) + 2u} = -e$$

$$-p = -3eu\left(\frac{\sqrt{3}}{2}\right) - 2eu$$

$$2p = 3\sqrt{3}\,eu + 4eu \quad : \mathrm{I}$$

LCM

$$3u\cos 30(3m) + (-2u)(2m) = 0 + p(m)$$

$$p = 9u(\sqrt{3}/2) - 4u$$

$$p = \frac{9}{2}\sqrt{3}u - 4u$$

$$2p = 9\sqrt{3}u - 8u \quad : \mathrm{II}$$

Equating I & II:

$$3\sqrt{3}e\cancel{u} + 4e\cancel{u} = 9\sqrt{3}\cancel{u} - 8\cancel{u}$$

$$\frac{e(3\sqrt{3} + 4)}{3\sqrt{3}+4} = \frac{9\sqrt{3} - 8}{3\sqrt{3}+4}$$

$$e = \frac{9\sqrt{3} - 8}{3\sqrt{3} + 4} = \boxed{0 \cdot 83}$$

Q15.

Before	Mass	After
$u\vec{i}$	$2m$	$p\vec{i}$
$-2u\vec{i}$	m	$q\vec{i}$

NLR

$$\frac{p-q}{u+2u} = -e$$

$$p-q = -3eu \quad : \mathrm{I}$$

LCM

$$u(2m) + (-2u)(m) = p(2m) + q(m)$$

$$2p + q = 0$$

$$\Rightarrow q = -2p \quad \mathrm{II}$$

Put II into I:

$$p - (-2p) = -3eu$$

$$3p = -3eu$$

$$p = -eu$$

$$\Rightarrow q = -2(-eu)$$

$$= 2eu$$

$$KE_{Bef} = \tfrac{1}{2}(2m)(u)^2 + \tfrac{1}{2}(m)(-2u)^2$$

$$= mu^2 + 2mu^2$$

$$= 3mu^2 = E$$

$$KE_{Aft} = \tfrac{1}{2}(2m)(-eu)^2 + \tfrac{1}{2}(m)(2eu)^2$$

$$= me^2u^2 + 2me^2u^2$$

$$= 3me^2u^2 = F$$

$$\sqrt{\frac{F}{E}} = \sqrt{\frac{3mu^2e^2}{3mu^2}} = \sqrt{\frac{e^2}{1}} = \boxed{e}$$

Q.E.D.

Q16.

Before	Mass	After
$u\vec{i}$	$2m$	$p\vec{i}$
$0\vec{i}$	m	$q\vec{i}$

i)

NLR

$$\frac{p-q}{u} = -e$$

$$p - q = -eu \quad : \mathrm{I}$$

LCM

$$u(2m) + 0 = p(2m) + q(m)$$

$$2p + q = 2u \quad \mathrm{II}$$

Solving I & II:

$$p - \cancel{q} = -eu$$
$$2p + \cancel{q} = 2u$$
$$3p = 2u - eu$$
$$p = \frac{u(2-e)}{3} = \boxed{\tfrac{1}{3}(2-e)u}$$

Q.E.D.

ii) Solving I & II:

$$\mathrm{I} \times 2: \quad \cancel{2p} - 2q = -2eu$$
$$\mathrm{II}: \quad (-)\cancel{2p} \overset{(-)}{+} q = (-)2u$$
$$-3q = -2eu - 2u$$
$$3q = 2eu + 2u$$
$$q = \frac{2u(e+1)}{3}$$

$$KE_{Bef} = \tfrac{1}{2}(2m)(u)^2 = mu^2$$
$$KE_{Aft} = \tfrac{1}{2}(2m)\left(\frac{u(2-e)}{3}\right)^2 + \tfrac{1}{2}(m)\left(\frac{2u(1+e)}{3}\right)^2$$
$$= mu^2\left(\frac{4+e^2-4e}{9}\right) + 2mu^2\left(\frac{1+2e+e^2}{9}\right)$$
$$= \frac{4mu^2 + mu^2e^2 - 4emu^2 + 2mu^2 + 4emu^2 + 2mu^2e^2}{9}$$
$$= \frac{2mu^2 + mu^2e^2}{3}$$

$$\text{Loss} = \frac{3mu^2}{3} - \left(\frac{2mu^2 + mu^2e^2}{3}\right) = \frac{mu^2 - mu^2e^2}{3}$$
$$= \boxed{\frac{mu^2(1-e^2)}{3}}$$

Q.E.D.

Q17.

Before	Mass	After
$\frac{v\sqrt{3}}{2}\vec{i} + \frac{v}{2}\vec{j}$	M	$p\vec{i} + \frac{v}{2}\vec{j}$
$0\vec{i} + 0\vec{j}$	$2M$	$q\vec{i} + 0\vec{j}$

$v\cos 30 = \frac{v\sqrt{3}}{2}$

$v\sin 30 = \frac{v}{2}$

NLR

$$\frac{p-q}{v\sqrt{3}/2} = -e$$

$$2p - 2q = -ev\sqrt{3} \quad : \mathrm{I}$$

LCM

$$\frac{v\sqrt{3}}{2}(M) + 0 = p(M) + q(2M)$$

$$p + 2q = \frac{v\sqrt{3}}{2}$$

$$2p + 4q = v\sqrt{3} \quad : \mathrm{II}$$

Solving I & II:

$$\mathrm{I} \times 2: \quad 4p - \cancel{4q} = -2ev\sqrt{3}$$

$$\mathrm{II}: \quad 2p + \cancel{4q} = v\sqrt{3}$$

$$6p = v\sqrt{3}(1-2e)$$

$$p = \frac{v\sqrt{3}(1-2e)}{6}$$

$$\text{Slope of P before} = \frac{\vec{j}\text{ comp}}{\vec{i}\text{ comp}} = \frac{\frac{v}{2}}{\frac{v\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$\text{Slope of P after} = \frac{\vec{j}\text{ comp}}{\vec{i}\text{ comp}} = \frac{v/2}{\frac{v\sqrt{3}(1-2e)}{6}} = \frac{3}{\sqrt{3}(1-2e)}$$

These are $\perp \Rightarrow m_1 \times m_2 = -1$

$$\Rightarrow \frac{1}{\sqrt{3}}\left(\frac{3}{\sqrt{3}(1-2e}\right) = -1$$

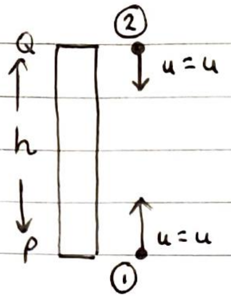
$$\Rightarrow \frac{1}{1-2e} = \frac{-1}{1}$$

$$\Rightarrow 2e - 1 = 1$$

$$2e = 2$$

$$\boxed{e = 1}$$

Q18.



Let T = time to collide

Particle ①

$u = u \quad a = -g \quad t = T$

$S_1 = uT + \frac{1}{2}(-g)T^2$

$S_1 = uT - \frac{1}{2}gT^2$

Particle ②

$u = u \quad a = g \quad t = T$

$S_2 = uT + \frac{1}{2}gT^2$

$$S_1 + S_2 = h$$

$$uT - \cancel{\tfrac{1}{2}gT^2} + uT + \cancel{\tfrac{1}{2}gT^2} = h$$

$$2uT = h$$

$$T = h/2u$$

Speed of both particles after $h/2u$ secs:

$V_1 = u - g(h/2u)$ $\qquad$ $V_2 = u + g(h/2u)$

$V_1 = u - \frac{gh}{2u}$ $\qquad$ $V_2 = u + \frac{gh}{2u}$

$V_1 = \frac{2u^2 - gh}{2u}$ $\qquad$ $V_2 = \frac{2u^2 + gh}{2u}$

LCM:

$$\cancel{m}\left(\frac{2u^2 - gh}{2u}\right) + \cancel{m}\left(\frac{-2u^2 - gh}{2u}\right) = 2\cancel{m}(v)$$

$$\frac{\cancel{2u^2} - gh - \cancel{2u^2} - gh}{2u} = 2v$$

$$\frac{-2gh}{2u} = 2v$$

$$\frac{-gh}{u} = 2v$$

$$\frac{-gh}{2u} = v$$

$$KE_{Bef} = \frac{1}{2}(m)\left(\frac{2u^2 - gh}{2u}\right)^2 + \frac{1}{2}(m)\left(\frac{2u^2 + gh}{2u}\right)^2$$

$$= \frac{m}{2}\left(\frac{4u^4 + g^2h^2 - 4u^2gh}{4u^2}\right) + \frac{m}{2}\left(\frac{4u^4 + g^2h^2 + 4u^2gh}{4u^2}\right)$$

$$= \frac{m}{2}\left(\frac{8u^4 + 2g^2h^2}{4u^2}\right)$$

$$KE_{Aft} = \frac{1}{2}(2m)\left(\frac{-gh}{2u}\right)^2$$
$$= \frac{mg^2h^2}{4u^2}$$

$$KE_{Loss} = KE_{Bef} - KE_{Aft}$$
$$= \frac{m}{2}\left(\frac{8u^4 + 2g^2h^2}{4u^2}\right) - \frac{mg^2h^2}{4u^2}$$
$$= \frac{m}{2}\left(\frac{8u^4 + 2g^2h^2}{4u^2}\right) - \frac{m}{2}\left(\frac{2g^2h^2}{4u^2}\right)$$
$$= \frac{m}{2}\left(\frac{8u^4 + \cancel{2g^2h^2} - \cancel{2g^2h^2}}{4u^2}\right)$$
$$= \frac{m}{\cancel{2}}\left(\frac{\overset{4}{\cancel{8}}u^4}{4u^2}\right)$$
$$= \boxed{mu^2}$$ Q.E.D.

$u = {}^{-gh}/_{2u}$ $a = g$ $v = ?$ $s = ?$

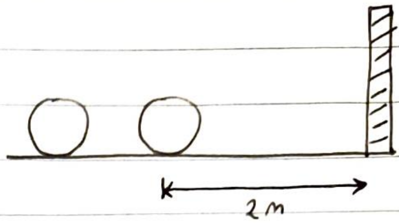
<u>To find s</u> (height of collision)

$$S_1 = uT - \frac{1}{2}gT^2$$
$$= u\left({}^{h}/_{2u}\right) - \frac{g}{2}\left(\frac{h}{2u}\right)^2$$
$$= \frac{h}{2} - \frac{g}{2}\left(\frac{h^2}{4u^2}\right)$$
$$= \frac{h}{2} - \frac{gh^2}{8u^2}$$
$$= \frac{4hu^2 - gh^2}{8u^2}$$

$$v^2 = u^2 + 2as$$
$$v^2 = \left({}^{-gh}/_{2u}\right)^2 + \cancel{2}g\left(\frac{4hu^2 - gh^2}{\overset{4}{\cancel{8}}u^2}\right)$$
$$v^2 = \frac{g^2h^2}{4u^2} + \frac{4ghu^2 - g^2h^2}{4u^2}$$
$$v^2 = \frac{\cancel{g^2h^2} + 4ghu^2 - \cancel{g^2h^2}}{4u^2}$$
$$v^2 = gh$$
$$\Rightarrow v = \boxed{\sqrt{gh}}$$ Q.E.D.

Q19.

Before	Mass	After
$u\vec{i}$	m	$p\vec{i}$
$0\vec{i}$	m	$q\vec{i}$



NLR

$$\frac{p-q}{u} = -e$$

$$p-q = -eu \quad : \mathrm{I}$$

LCM

$$m(u) + 0 = m(p) + m(q)$$

$$p+q = u \quad : \mathrm{II}$$

Solving I & II:

$$\begin{aligned} p - \cancel{q} &= -eu \\ p + \cancel{q} &= u \\ \hline 2p &= u - eu \\ p &= \frac{u(1-e)}{2} \\ p &= \frac{u-eu}{2} \end{aligned}$$

$$\begin{aligned} \cancel{p} - q &= -eu \\ (-)\,p \overset{(-)}{+} q &= (-)\,u \\ \hline -2q &= -u - eu \\ q &= \frac{u(1+e)}{2} \\ q &= \frac{u+eu}{2} \end{aligned}$$

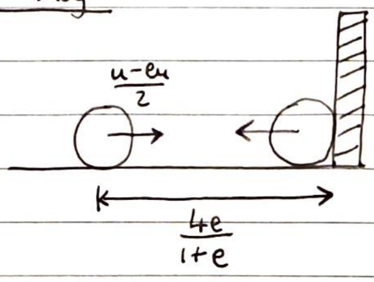
Time taken for 2^{nd} sphere to hit wall $= 2 \big/ \frac{u+eu}{2} = \frac{4}{u+eu}$

Dist travelled by sphere 1 in that time $= \left(\frac{u-eu}{2}\right)\left(\frac{4}{u+eu}\right)$

$$= \frac{2u(1-e)}{u(1+e)}$$

$$= \frac{2-2e}{1+e}$$

New Diag:



$\Rightarrow$ Dist between spheres

$$= 2 - \frac{2-2e}{1+e} = \frac{2+2e-2+2e}{1+e}$$

$$= 4e/1+e$$

Vel 2^{nd} sphere after striking wall $= -e\left(\frac{u+eu}{2}\right)$

$$= \frac{-eu-e^2u}{2}$$

$$\Rightarrow \text{Speed} = \frac{eu+e^2u}{2}$$

Let T = time to collide again

Dist travelled by Sph ① $= \left(\frac{u-eu}{2}\right)(T)$

Dist travelled by Sph ② $= \left(\frac{eu+e^2u}{2}\right)(T)$

$$\Rightarrow \left(\frac{u-eu}{2}\right)(T) + \left(\frac{eu+e^2u}{2}\right)(T) = \frac{4e}{1+e}$$

$$T\left(\frac{u-\cancel{eu}+\cancel{eu}+e^2u}{2}\right) = \frac{4e}{1+e}$$

$$T = \frac{8e}{(1+e)(u+e^2u)}$$

<u>To find dist from wall:</u>

Dist = Speed of Sph ② $\times$ T

$$= \frac{eu+e^2u}{\cancel{2}}\left(\frac{\overset{4}{\cancel{8}}e}{(1+e)(u+e^2u)}\right)$$

$$= \frac{4e(eu+e^2u)}{(1+e)(u+e^2u)}$$

$$= \frac{4e^2\cancel{u}(\cancel{1+e})}{(\cancel{1+e})\cancel{u}(1+e^2)}$$

$$= \boxed{\frac{4e^2}{1+e^2}}$$

Q.E.D.

2012 Q5

Q5 a) $A \to B$

Before	Mass	After
$5\vec{i}$	$3m$	$p\vec{i}$
$0\vec{i}$	$2m$	$q\vec{i}$

NLR

$$\frac{p-q}{5} = -e$$

$$p - q = -5e \quad \text{I}$$

LCM

$$15m + 0 = 3mp + 2mq$$

$$3p + 2q = 15 \quad \text{II}$$

Solving I & II:

$\text{I}\times 2$: $2p - \cancel{2q} = -10e$

II: $3p + \cancel{2q} = 15$

$$5p = 15 - 10e$$

$$p = 3 - 2e$$

Using I:

$$3 - 2e - q = -5e$$

$$q = 3 + 3e$$

$B \to C$

Before	Mass	After
$(3+3e)\vec{i}$	$2m$	$r\vec{i}$
$0\vec{i}$	m	$s\vec{i}$

NLR

$$\frac{r-s}{3+3e} = -e$$

$$r - s = -3e - 3e^2$$

LCM

$$(6+6e)m = 2mr + sm$$

$$2r + s = 6 + 6e$$

Solving:

$r - \cancel{s} = -3e - 3e^2$

$2r + \cancel{s} = 6 + 6e$

$$3r = -3e^2 + 3e + 6$$

$$r = \boxed{-e^2 + e + 2}$$

Solving same eqns for s gives:

$$s = \boxed{2e^2 + 4e + 2}$$

$s > r \Rightarrow$ C will move away from B $\Rightarrow$ no more collisions between them

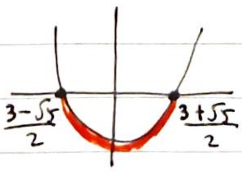
No more collisions between A and B as long as $p < r$

$$3 - 2e < -e^2 + e + 2$$

$$e^2 - 3e + 1 < 0$$

$a = 1 \quad b = -3 \quad c = 1$

$$e = \frac{3 \pm \sqrt{5}}{2}$$



$$\Rightarrow \frac{3-\sqrt{5}}{2} < e < \frac{3+\sqrt{5}}{2}$$

$$\Rightarrow \boxed{e > \frac{3-\sqrt{5}}{2}}$$

Q.E.D.

b)

Before	Mass	After
$u\cos\alpha\,\vec{i} + u\sin\alpha\,\vec{j}$	m	$p\vec{i} + u\sin\alpha\,\vec{j}$
$0\vec{i} + 0\vec{j}$	m	$q\vec{i} + 0\vec{j}$

<u>NLR</u>

$$\frac{p-q}{u\cos\alpha} = \frac{-1}{3}$$

$$3p - 3q = -u\cos\alpha \quad \text{I}$$

<u>LCM</u>

$$u\cos\alpha(m) + 0 = p(m) + q(m)$$

$$p + q = u\cos\alpha \quad \text{II}$$

<u>Solving I & II:</u>

$$\text{I}: \quad 3p - \cancel{3q} = -u\cos\alpha$$

$$\text{II} \times 3: \quad 3p + \cancel{3q} = 3u\cos\alpha$$

$$6p = 2u\cos\alpha$$

$$p = \frac{u\cos\alpha}{3}$$

$$\text{Slope before} = \frac{\vec{j}\text{ comp}}{\vec{i}\text{ comp}} = \frac{u\sin\alpha}{u\cos\alpha} = \tan\alpha = m_1$$

$$\text{Slope after} = \frac{u\sin\alpha}{\frac{u\cos\alpha}{3}} = 3\tan\alpha = m_2$$

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \frac{\tan\alpha - 3\tan\alpha}{1 + \tan\alpha(3\tan\alpha)}$$

$$= \left| \frac{-2\tan\alpha}{1 + 3\tan^2\alpha} \right|$$

$$= \boxed{\frac{2\tan\alpha}{1 + 3\tan^2\alpha}}$$

Q.E.D.

2014 Q5

Q5 a) i)

Before	Mass	After
$u\vec{i}$	$2m$	$p\vec{i}$
$0\vec{i}$	$7m$	$q\vec{i}$

NLR

$$\frac{p-q}{u} = -\frac{1}{2}$$

$$2p - 2q = -u \quad : \mathrm{I}$$

LCM

$$u(2m) + 0 = p(2m) + q(7m)$$

$$2p + 7q = 2u \quad \mathrm{II}$$

Solving I & II:

I: $\cancel{2p} - 2q = -u$

II: $(-)\cancel{2p} \overset{(-)}{+} 7q \overset{(-)}{=} 2u$

$$-9q = -3u$$

$$q = \frac{u}{3}$$

Using I:

$$2p - 2\left(\frac{u}{3}\right) = -u$$

$$6p = -u$$

$$p = \frac{-u}{6}$$

As A rebounds in opposite direction and even after B hits the wall its speed coming back will only match A's $\Rightarrow$ no further collision.

ii) $KE_{Before} = \frac{1}{2}(2m)(u)^2 + 0 = mu^2$

$KE_{After} = \frac{1}{2}(2m)(p)^2 + \frac{1}{2}(7m)(q)^2 = \frac{mu^2}{36} + \frac{14mu^2}{36} = \frac{15mu^2}{36} = \frac{5mu^2}{12}$

$$\Rightarrow KE_{Loss} = mu^2 - \frac{5mu^2}{12} = \frac{7mu^2}{12}$$

Collision of B with wall:

$KE\ Before = \frac{1}{2}(7m)\left(\frac{u}{3}\right)^2 = \frac{7mu^2}{18}$

$KE\ After = \frac{1}{2}(7m)\left(-\frac{u}{6}\right)^2 = \frac{7mu^2}{72}$

$\Rightarrow Loss = \frac{28mu^2}{72} - \frac{7mu^2}{72} = \frac{7mu^2}{24}$

$$\Rightarrow \text{Total Loss} = \frac{7mu^2}{12} + \frac{7mu^2}{24} = \boxed{\frac{7mu^2}{8}}$$

b)

i)

Before	Mass	After
$3u\vec{i} + 4u\vec{j}$	$2m$	$p\vec{i} + 4u\vec{j}$
$-4u\vec{i} + 3u\vec{j}$	m	$q\vec{i} + 3u\vec{j}$

NLR

$$\frac{p-q}{7u} = -e$$

$$p - q = -7eu \quad : \mathrm{I}$$

LCM

$$3u(2m) + (-4u)(m) = (p)(2m) + (q)(m)$$

$$2p + q = 2u \quad : \mathrm{II}$$

Solving I & II:

$$\mathrm{I}: \; p - \cancel{q} = -7eu$$

$$\mathrm{II}: \; 2p + \cancel{q} = 2u$$

$$3p = 2u - 7eu$$

$$p = \frac{2u - 7eu}{3}$$

Solving I & II:

$$\mathrm{I} \times 2: \; \cancel{2p} - 2q = -14eu$$

$$\mathrm{II}: \; (-1)\cancel{2p} \,(-1)+\, q = (-1)\, 2u$$

$$-3q = -14eu - 2u$$

$$q = \frac{2u + 14eu}{3}$$

$$KE_{Before} = \tfrac{1}{2}(2m)(3u)^2 + \tfrac{1}{2}(m)(-4u)^2 = 9mu^2 + 8mu^2 = 17mu^2$$

$$KE_{After} = \tfrac{1}{2}(2m)\left(\frac{u(2-7e)}{3}\right)^2 + \tfrac{1}{2}(m)\left(\frac{2u(1+7e)}{3}\right)^2$$

$$= mu^2\left(\frac{49e^2 - 28e + 4}{9}\right) + 2mu^2\left(\frac{49e^2 + 14e + 1}{9}\right)$$

$$= \frac{mu^2}{9}\left(6 + 147e^2\right)$$

$$\Rightarrow KE_{Loss} = 17mu^2 - \frac{mu^2}{9}\left(6 + 147e^2\right)$$

$$= \frac{153mu^2 - 6mu^2 - 147mu^2e^2}{9}$$

$$= \frac{147mu^2 - 147mu^2e^2}{9} = \frac{25mu^2}{2}$$

$$\Rightarrow 294 - 294e^2 = 225$$

$$294e^2 = 69$$

$$e^2 = \frac{69}{294}$$

$$\Rightarrow e = \boxed{0.484}$$

ii) $\Rightarrow p = \frac{2u - 7(0.484)u}{3} = -0.463u$

$$I = m\vec{v} - m\vec{u} = 2m(-0.463u) - 2m(3u) = -6.93mu$$

$$\Rightarrow k = \boxed{6.93}$$

2017 Q5

Q5 a) i)

Before	Mass	After
$6\vec{i}$	1.5	$-v\vec{i}$
$0\vec{i}$	m	$2v\vec{i}$

This couldn't be the (−) one

NLR

$$\frac{-v-2v}{6} = -e$$

$$-3v = -6e$$

$$v = 2e \quad : \mathrm{I}$$

LCM

$$6(1.5) + 0 = 1.5(-v) + m(2v)$$

$$2mv - 1.5v = 9 \quad : \mathrm{II}$$

$$KE_A \text{ Before} = \tfrac{1}{2}(1.5)^2(6)^2 = 27$$

$$KE_A \text{ After} = \tfrac{1}{2}(1.5)(v)^2 = 0.75v^2$$

$$\Rightarrow \text{Loss in KE} = 27 - 0.75v^2$$

$$80\% \text{ of KE Loss} = 21.6 - 0.6v^2$$

$$KE_B \text{ Before} = \tfrac{1}{2}(m)(0)^2 = 0$$

$$KE_B \text{ After} = \tfrac{1}{2}(m)(2v)^2 = 2v^2m$$

$$\Rightarrow \text{KE GAIN} = 2v^2m = 21.6 - 0.6v^2$$

$$\Rightarrow m = \frac{21.6 - 0.6v^2}{2v^2}$$

Using II:

$$\cancel{2}\cancel{v}\left(\frac{21.6 - 0.6v^2}{\cancel{2}v^{\cancel{2}}}\right) - 1.5v = 9$$

$$21.6 - 0.6v^2 - 1.5v^2 = 9v$$

$$-2.1v^2 - 9v + 21.6 = 0$$

$$2.1v^2 + 9v - 21.6 = 0$$

$a = 2.1 \quad b = 9 \quad c = -21.6$

$$\Rightarrow v = \frac{9 \pm \sqrt{(9)^2 - 4(2.1)(-21.6)}}{2(2.1)} = \boxed{\tfrac{12}{7}\text{ m/s}} \text{ or } \cancel{-6\text{ m/s}}$$

ii) Using I: $e = \frac{v}{2} = \frac{12/7}{2} = \boxed{\frac{6}{7}}$

b)

	Before	Mass	After
i)	$u\cos\alpha\,\vec{i} + u\sin\alpha\,\vec{j}$	$3m$	$p\vec{i} + u\sin\alpha\,\vec{j}$
	$0\vec{i} + 0\vec{j}$	$7m$	$v\vec{i} + 0\vec{j}$

$e = 2/7$

NLR

$$\frac{p - v}{u\cos\alpha} = \frac{-2}{7}$$

$$7p - 7v = -2u\cos\alpha \quad : \text{I}$$

LCM

$$u\cos\alpha(3m) + 0 = p(3m) + v(7m)$$

$$3p + 7v = 3u\cos\alpha \quad : \text{II}$$

Solving I & II:

$$\text{I}: \quad 7p - \cancel{7v} = -2u\cos\alpha$$

$$\text{II}: \quad 3p + \cancel{7v} = 3u\cos\alpha$$

$$10p = u\cos\alpha$$

$$p = \frac{u\cos\alpha}{10}$$

Using I:

$$7\left(\frac{u\cos\alpha}{10}\right) - 7v = -2u\cos\alpha$$

$$7u\cos\alpha - 70v = -20u\cos\alpha$$

$$27u\cos\alpha = 70v$$

$$\Rightarrow u = \boxed{\frac{70v}{27\cos\alpha}}$$

ii) Slope before $= \dfrac{\vec{j}\text{ comp}}{\vec{i}\text{ comp}} = \dfrac{u\sin\alpha}{u\cos\alpha} = \tan\alpha = \tan 30° = \dfrac{1}{\sqrt{3}} = m_1$

Slope after $= \dfrac{u\sin\alpha}{p} = \dfrac{u\sin\alpha}{\frac{u\cos\alpha}{10}} = 10\tan\alpha = 10\tan 30 = \dfrac{10}{\sqrt{3}} = m_2$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{1}{\sqrt{3}} - \frac{10}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)\left(\frac{10}{\sqrt{3}}\right)}\right|$$

$$= \left|\frac{-9/\sqrt{3}}{13/3}\right|$$

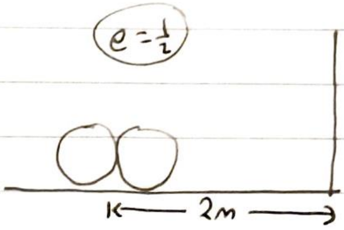
$$\tan\theta = \frac{9\sqrt{3}}{13}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{9\sqrt{3}}{13}\right) = \boxed{50.2°}$$

2019 Q5

Q5 a) i)

	Before	Mass	After
A:	$u\vec{i}$	$3m$	$p\vec{i}$
B:	$-u\vec{i}$	m	$q\vec{i}$



NLR

$$\frac{p-q}{2u} = \frac{-1}{2}$$

$$2p - 2q = -2u$$

$$p - q = -u \; : \text{I}$$

LCM

$$u(3m) + (-u)(m) = p(3m) + q(m)$$

$$3p + q = 2u \; : \text{II}$$

Solving I & II:

I: $p - \cancel{q} = -u$

II: $3p + \cancel{q} = 2u$

$$4p = u$$

$$\boxed{p = u/4}$$

Using I:

$$u/4 - q = -u$$

$$q = u/4 + u$$

$$\boxed{q = 5u/4}$$

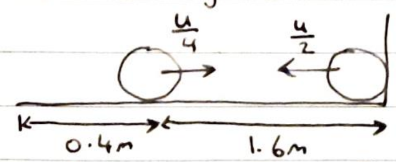
ii) Vel B before hitting wall $= 5u/4 \, \vec{i}$

$\Rightarrow$ Vel after hitting wall $= -\frac{2}{5}\left(\frac{5u}{4}\right) = -\frac{u}{2}\vec{i}$

Dist to wall = 2m

$\Rightarrow$ time to hit wall for B $= \frac{2}{5u/4} = \frac{8}{5u}$ secs

Dist travelled by A in that time $= \frac{u}{4}\left(\frac{8}{5u}\right) = \frac{2}{5} = 0.4m$



Let T = time to collide

Dist travelled by A $= \frac{uT}{4}$ Dist travelled by B $= \frac{uT}{2}$

$\Rightarrow \frac{uT}{4} + \frac{uT}{2} = 1.6$

$uT = 32/15$

$\Rightarrow T = 32/15u$

$\Rightarrow \frac{8}{5u} + \frac{32}{15u} = 4$

$\Rightarrow \frac{56}{15u} = 4$

$\Rightarrow 60u = 56$

$\Rightarrow u = \boxed{14/15 \text{ m/s}}$ or $\boxed{0.93 \text{ m/s}}$

b)

i)

Before	Mass	After
$3u\vec{i} + 4u\vec{j}$	$2m$	$p\vec{i} + 4u\vec{j}$
$-4u\vec{i} + 3u\vec{j}$	m	$q\vec{i} + 3u\vec{j}$

NLR

$$\frac{p-q}{7u} = \frac{-5}{7}$$

$$7p - 7q = -35u$$

$$p - q = -5u \quad : \text{I}$$

LCM

$$3u(2m) + (-4u)(m) = p(2m) + q(m)$$

$$2p + q = 2u \quad : \text{II}$$

Solving I & II:

$$p - \cancel{q} = -5u$$

$$2p + \cancel{q} = 2u$$

$$3p = -3u$$

$$p = -u$$

Using I:

$$-u - q = -5u$$

$$q = 4u$$

$\Rightarrow$ Speed P : $\sqrt{(-u)^2 + (4u)^2} = \boxed{u\sqrt{17} \text{ m/s}}$

Speed Q : $\sqrt{(4u)^2 + (3u)^2} = \boxed{5u \text{ m/s}}$

ii) Slope P after $= \dfrac{\vec{j} \text{ comp}}{\vec{i} \text{ comp}} = \dfrac{4u}{-u} = -4$

Slope Q after $= 3u/4u = 3/4$

$$\text{Tan}\,\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-4 - 3/4}{1 + (-4)(3/4)}\right|$$

$$= \left|\frac{-19/4}{-2}\right|$$

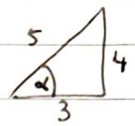
$$= |2.375|$$

$\Rightarrow \theta = \text{Tan}^{-1}(2.375) = \boxed{67.17°}$

Q2 (b)

2023 Q2(b)

Before	Mass	After
$\frac{12}{5}\vec{i} + \frac{16}{5}\vec{j}$	m	$p\vec{i} + \frac{16}{5}\vec{j}$
$0\vec{i} + 3.2\vec{j}$	$2m$	$q\vec{i} + 3.2\vec{j}$



$\cos\alpha = 3/5$

$\sin\alpha = 4/5$

$4\cos\alpha\,\vec{i} + 4\sin\alpha\,\vec{j}$

$\frac{12}{5}\vec{i} + \frac{16}{5}\vec{j}$

NLR

$$\frac{p-q}{\frac{12}{5}} = -e$$

$$5p - 5q = -12e \quad : \mathrm{I}$$

LCM

$$\frac{12}{5}(m) + 0 = p(m) + q(2m)$$

$$p + 2q = \frac{12}{5}$$

$$5p + 10q = 12 \quad : \mathrm{II}$$

Solving I & II:

I: $\cancel{5p} - 5q = -12e$

II: $(-)\ \cancel{5p} \overset{(-)}{+} 10q \overset{(-)}{=} 12$

$$-15q = -12e - 12$$

$$15q = 12e + 12$$

$$5q = 4e + 4$$

$$q = \frac{4(e+1)}{5}$$

$\Rightarrow$ Vel: $\boxed{\frac{4e+4}{5}\vec{i} + 3.2\vec{j}}$

Solving I & II:

I×2: $10p - \cancel{10q} = -24e$

II: $5p + \cancel{10q} = 12$

$$15p = 12 - 24e$$

$$5p = 4 - 8e$$

$$p = \frac{4-8e}{5}$$

$\Rightarrow$ Vel $= \boxed{\frac{4-8e}{5}\vec{i} + \frac{16}{5}\vec{j}}$